Clustering of Coronal Mass Ejections

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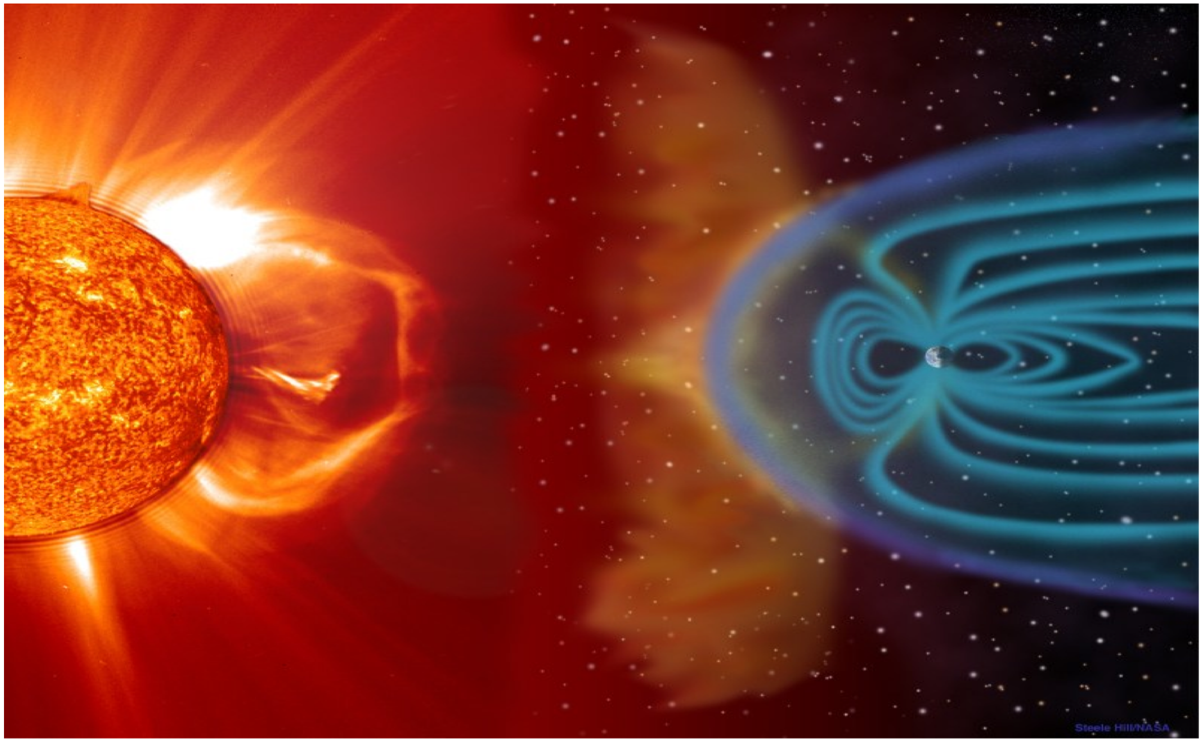
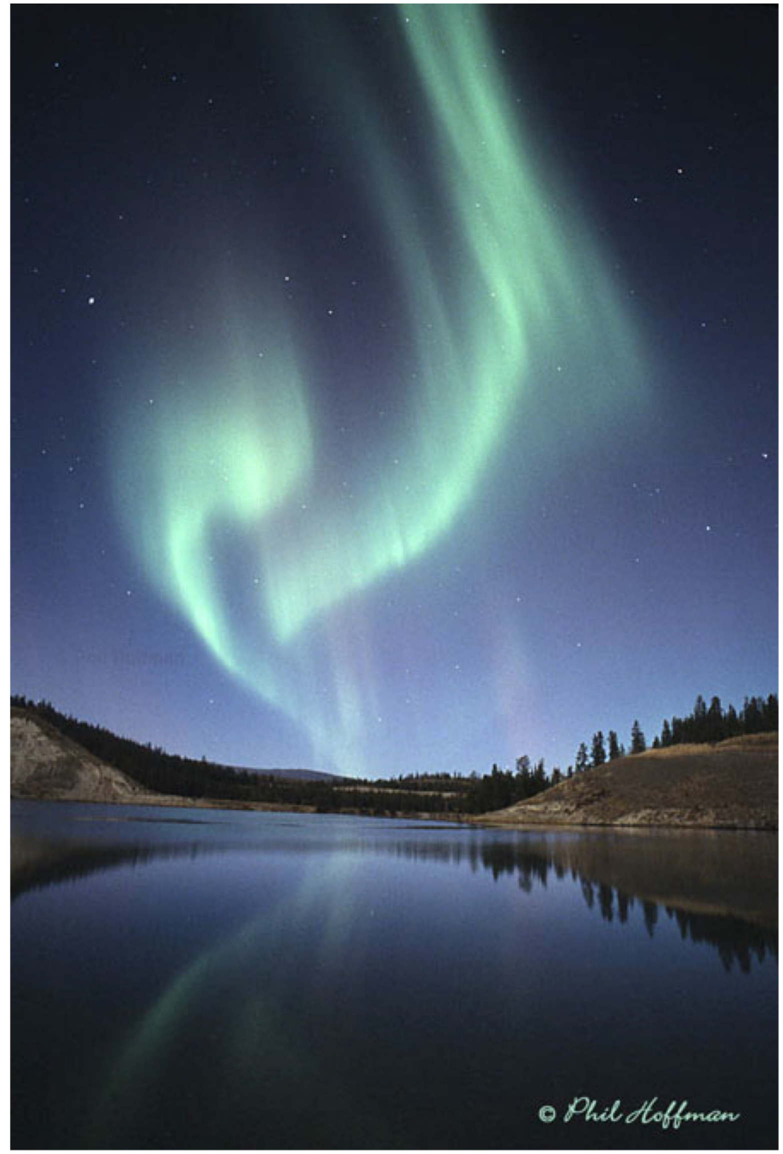
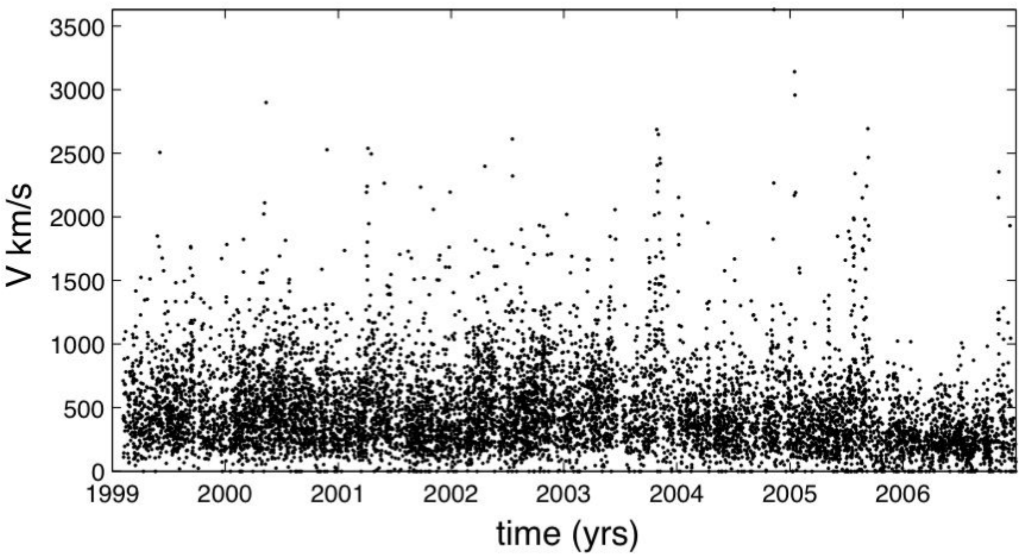


Fast Coronal Mass Ejections (CMEs) are critical drivers of Space Weather since they generate energetic particles and disturb the Earth magnetosphere triggering geomagnetic storms

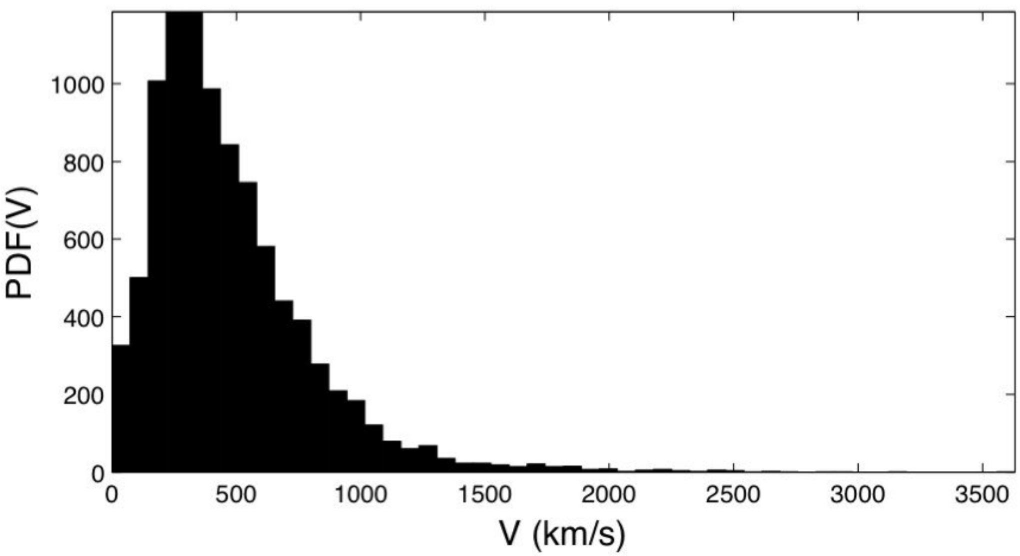


Statistics of Fast CMEs

Distribution Function of CME speeds



9,408 CMEs detected by SOHO LASCO in 1999-2006



Non-Gaussian PDF

Vmean = 472 km/s

Extremes:

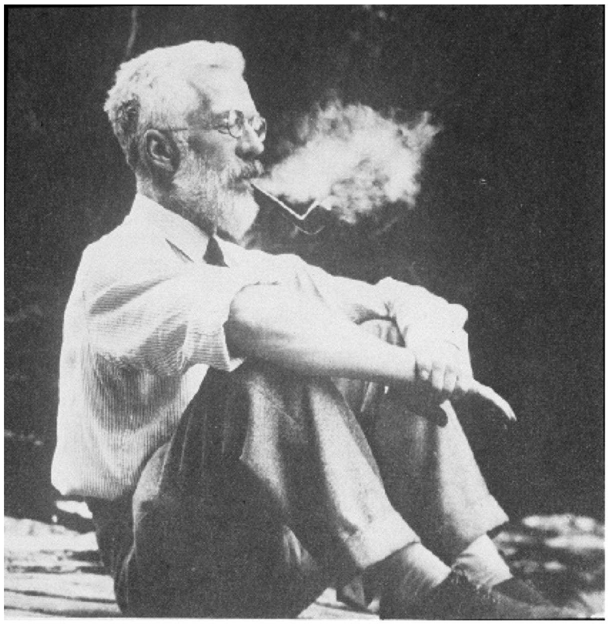
18% V > 700 km/s

6.2% V > 1,000 km/s

0.5% V > 2,000 km/s.

Statistics of Small Numbers
i.e. statistics of extreme (rare) events

Fisher-Tippett-Gnedenko Theorem of Extreme Value Theory



Ronald Fisher
(1890-1962)
Statistical Methods for Research Workers

Leonard Tippett (1902-1985)
British Cotton Industry random number table (now random number generators)

Boris Gnedenko
(1912-1995)
Textile Institute, Ivanovo

The Theorem

If e_1, e_2, …,e_k, … are iid random events and $M_n = \max(e_1, e_2, \ldots, e_n)$

Then Prob($M_n \le x$), as $n \rightarrow \infty$ is

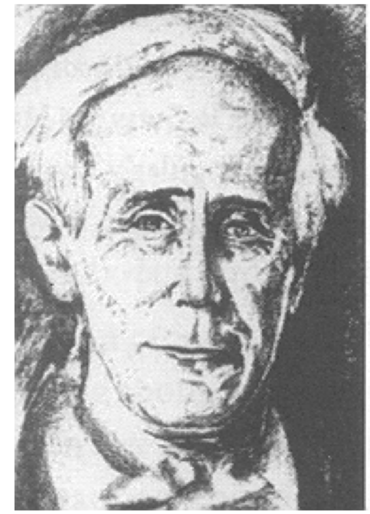
$G = \exp[-\exp(-x)]$ Gumbel (1891-1966)

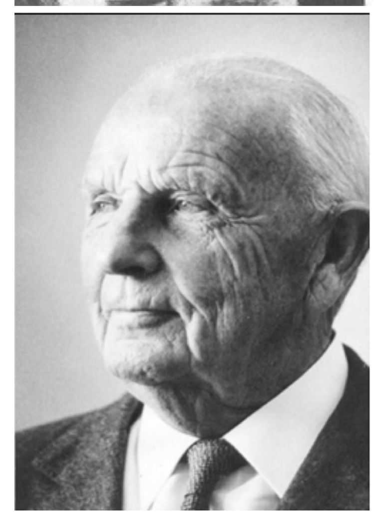
or

$F = \exp(-x^{-\alpha}) \approx 1 - x^{-\alpha}$ Frechet (1878-1973)

or

$W = 1 - \exp(-x^{\alpha})$ Weibull (1887-1979)





Great!

But there are no math justified procedure for curve fitting.

It depends on:

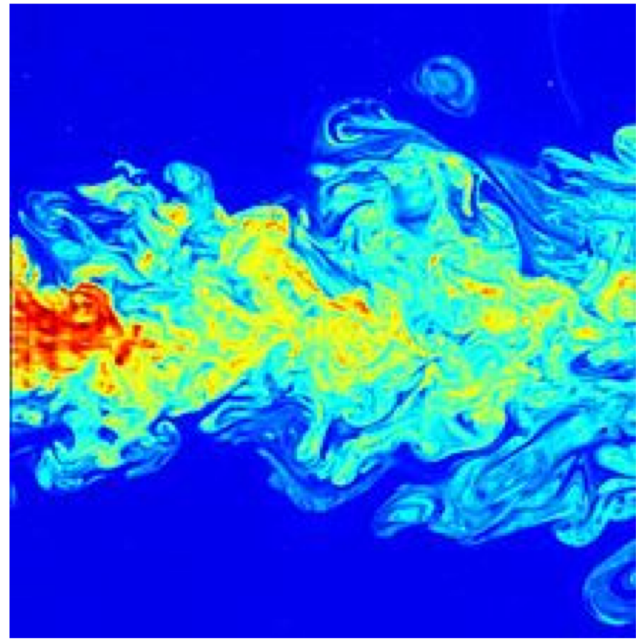
data sample,

adjustable parameters,

skill of a researcher.

Scaling Approach

A. N. Kolmogorov



$$\delta \mathbf{u}(r) = \mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x})$$

$$\langle [\delta \mathbf{u}(r)]^n \rangle = C_n \varepsilon^{n/3} r^{n/3}$$

Scaling Approach to Extremes: Max Spectrum

Consider time series of CMEs recorded in the solar corona, 1,2,...,N.

Divide time axis into progressively increasing blocks: $\Delta t = 2^j$, j =1,2 ,3,...

Find maxima M = max V(j) of CME speeds at each time scale.

Take log and average over number of intervals (k):

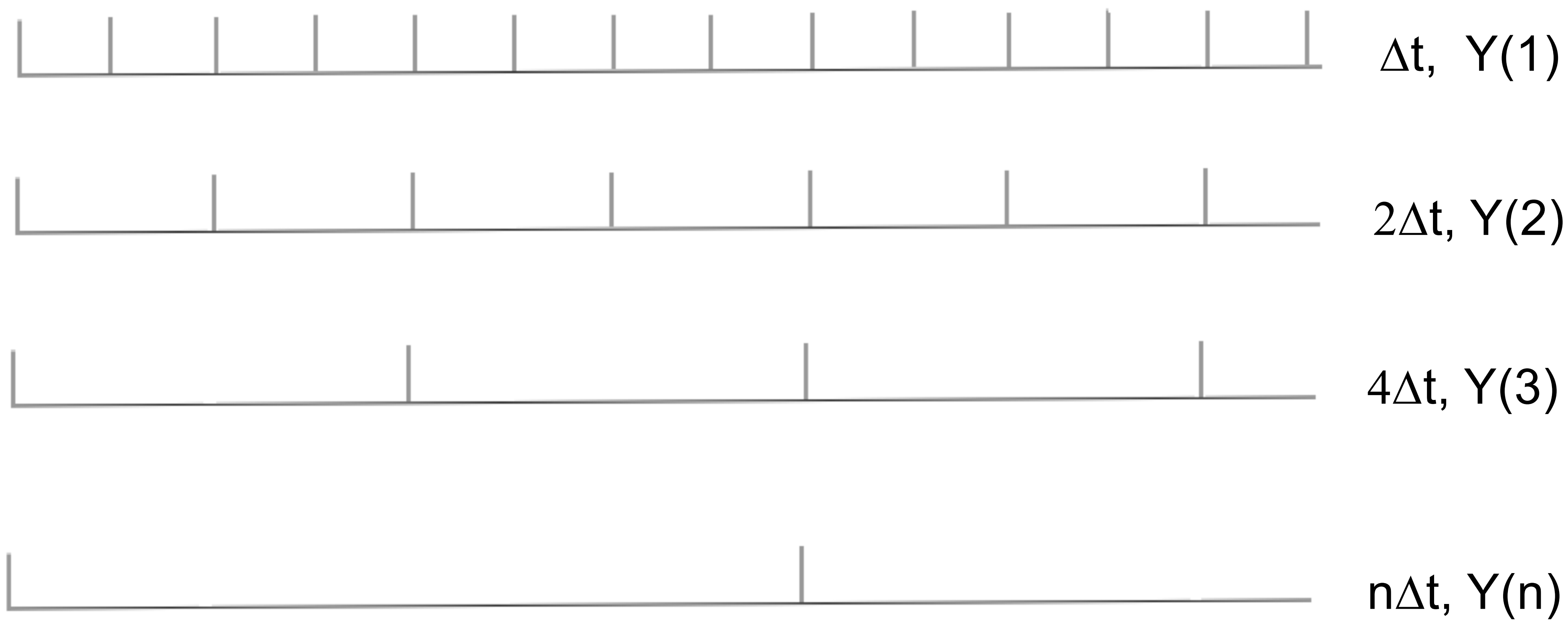
$$Y(j) = N^{-1} \sum_k \log_2 M(j,k) \quad \text{-- Max Spectrum}$$

The slope of max spectrum ($1/\alpha$) is a heavy–tail exponent of extreme value probability density

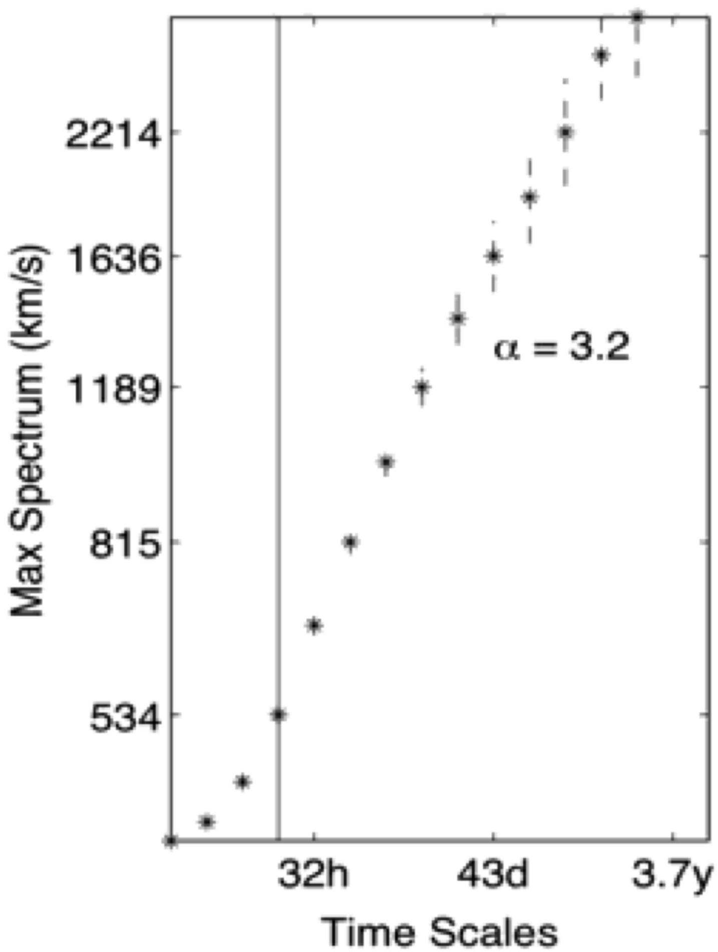
$$P = \exp(-Cx^{-\alpha}) \sim 1 - x^{-\alpha}, \text{ as } x \to \infty.$$

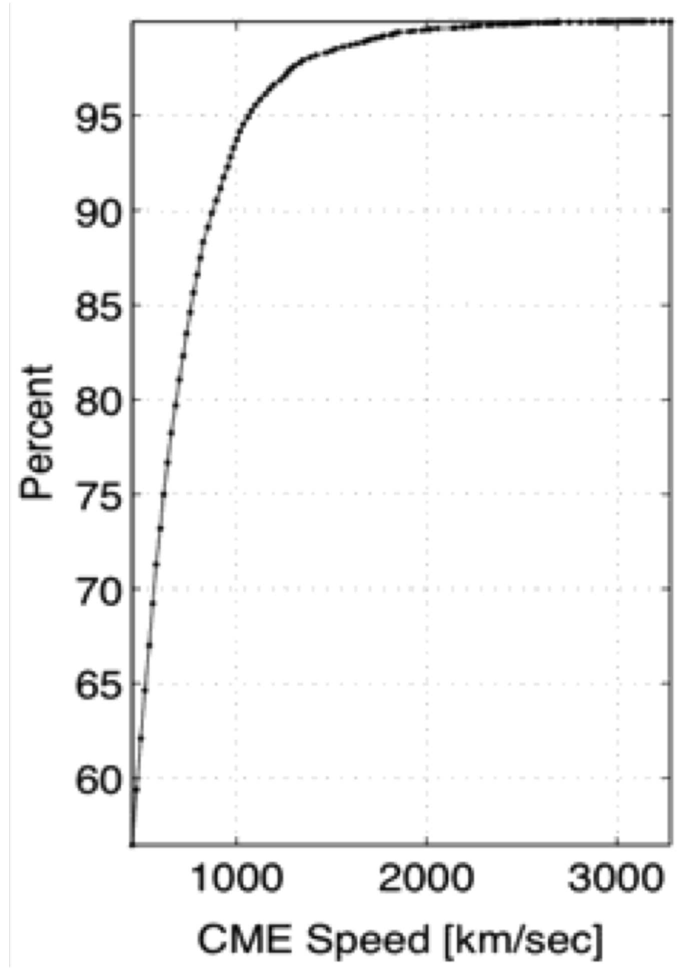
Stoev et al., 2006

Construction of Max Spectrum



CDF Tail of Fast CMEs

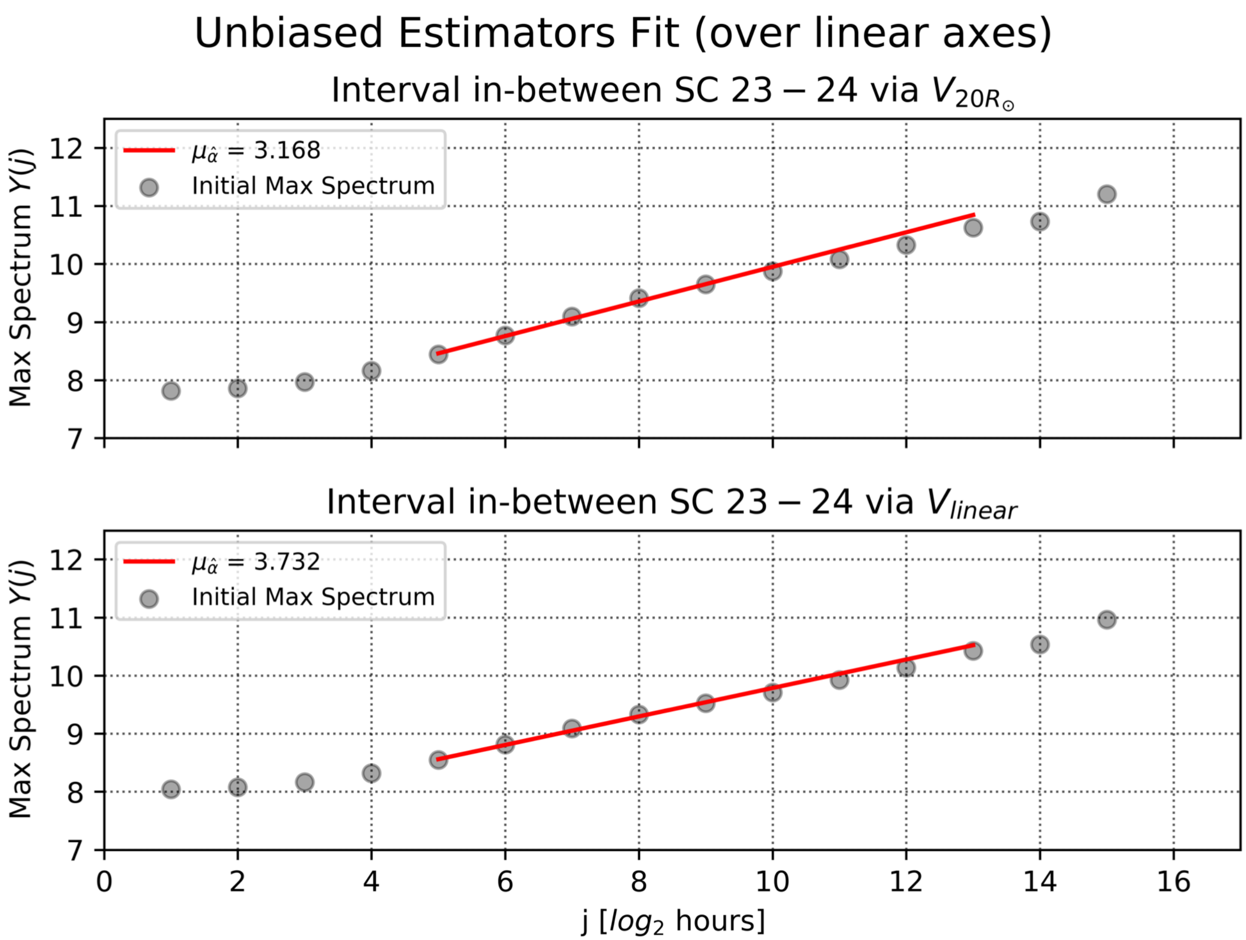




Range of speeds limited by linear fit gives a definition of “fast” CMEs.

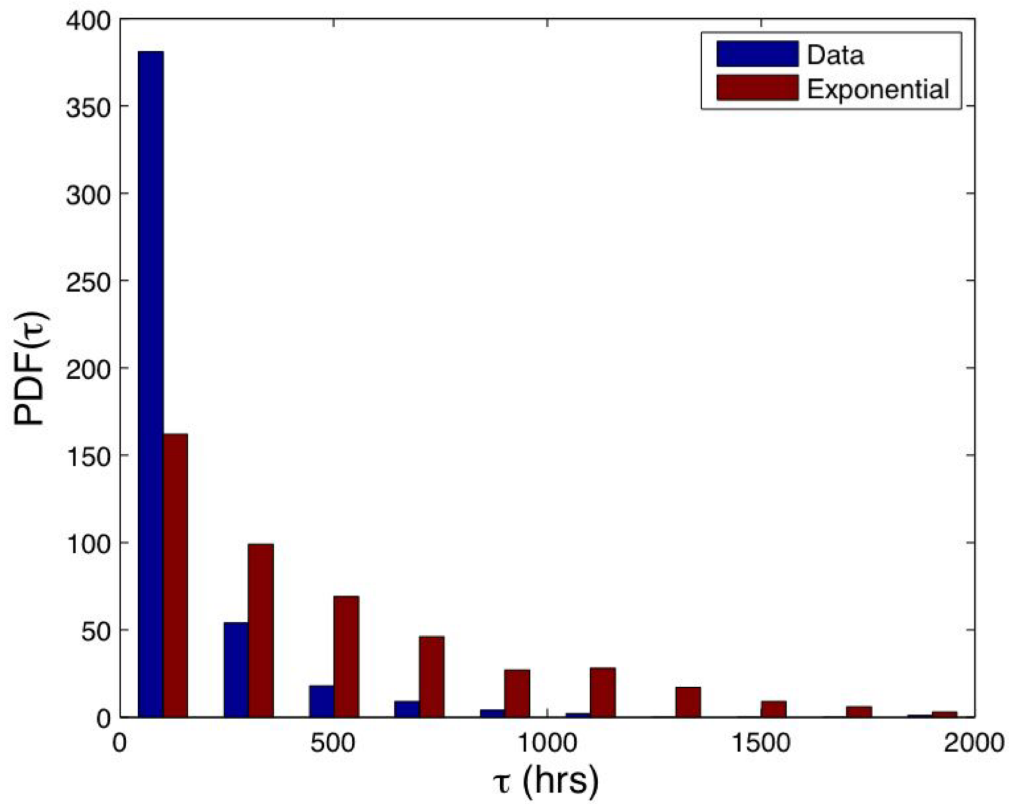
Cumulative distribution function. Its high-speed tail is $1 - V^{-\alpha}$.

Max Spectrum of Fast CMEs for cycles 23 and 24



Clustering of Fast CMEs in time

For a pure random (Poissonian) process the times between events $\tau = t(i+1) - t(i)$ are independent and exponentially distributed: $\exp(-\tau/\tau_0)$.
Observed fast CMEs (blue) are correlated in time i.e. arrive in clusters.



Clustering of extremes is characterized by the extremal index: $\exp(-\theta\ \tau/\tau_0)$.

$\theta = 1$ for non-clustered events.

Estimate of the Extremal Index

Max-Spectrum for independent extreme events (large j):

$$Y(j) \approx j/\alpha + C.$$

For dependent extreme events:

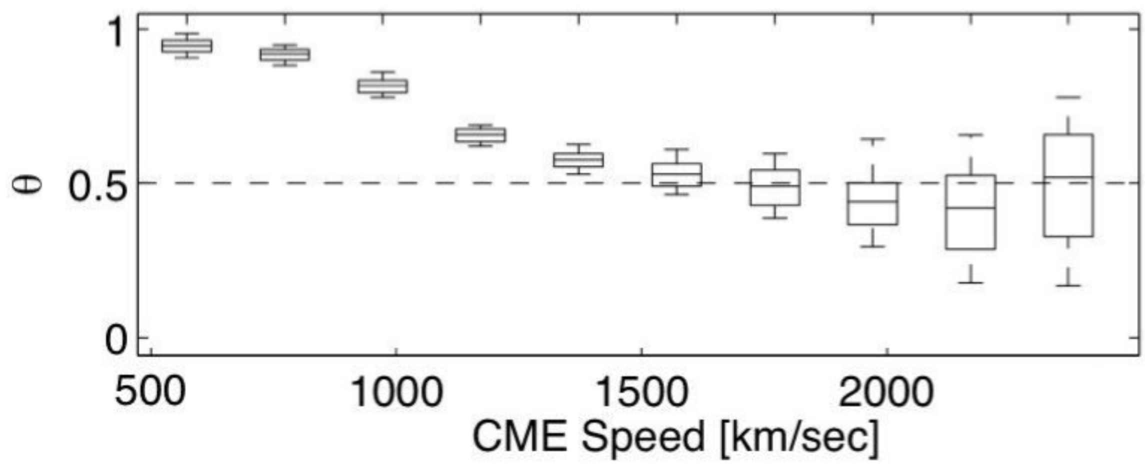
$$Y(j) \approx j/\alpha + C + \log_2(\theta)/\alpha ,$$

where $0 < \theta < 1$ is the extremal index.

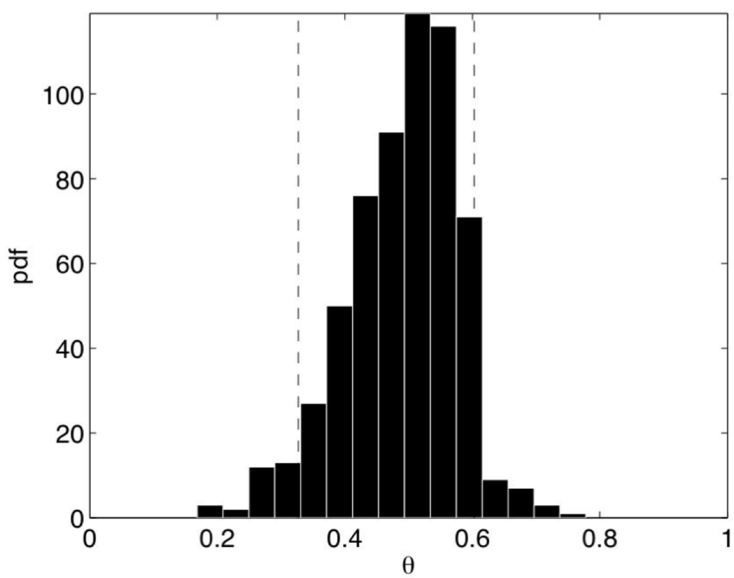
We estimate θ by subtracting the data Max-Spectrum from the Max-Spectrum of bootstrapped (uncorrelated) data.

Stoev et al (2006)

Extremal Index of Fast CMEs



Determined by comparing observed Max Spectrum with Max Spectrum simulated from randomized data



$$\langle\theta\rangle = 0.5 \quad (0.3 - 0.6)$$

Fast CMEs with speeds 1,000-2,000 km/s arrive in clusters, *on average* 2-3 events closely spaced in time.

Counting Clusters

1. Sort time intervals between extreme events

$$\tau_1 \leq \tau_2 \leq \tau_3 \ldots \leq \tau_c \leq \ldots \leq \tau_{n-1}$$

inter-cluster times intra-cluster times

$\tau_c = n\,\theta$ is 'de-clustering time' (Ferro & Segers, 2003)

2. Go along extreme events in real time

If 1, 2, .., m subsequent time intervals $\tau_i < \tau_c$ these events constitute a cluster of size m. If any $\tau_k > \tau_c$ the k and k+1 events belong to different clusters (or are singles).

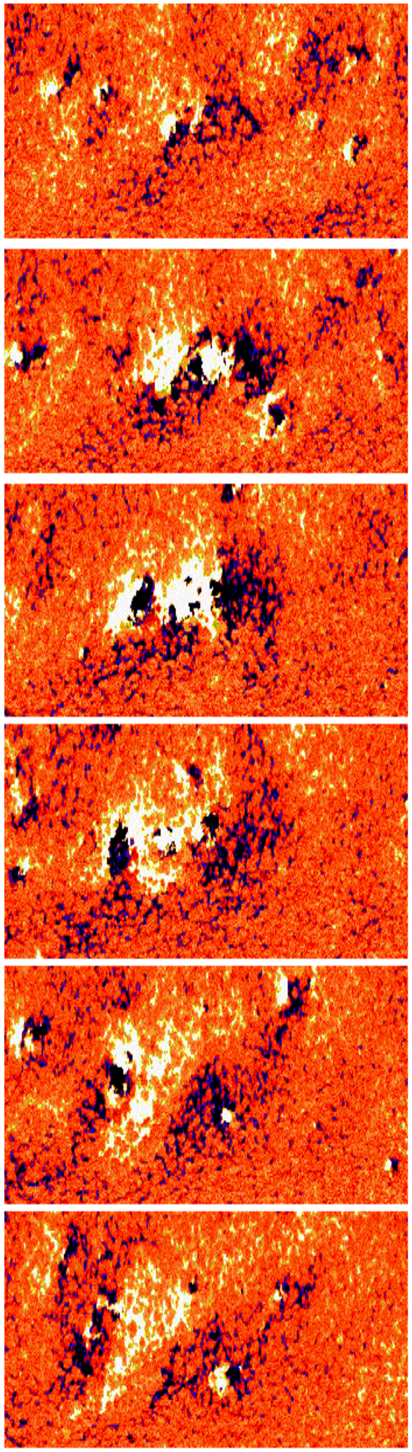
Statistics of Fast CME Clusters

Size	N of Clusters	N of CMEs in Clusters	Proportion %	Mean Duration (hrs)
1	177	177	61	–
2	53	106	18	20
3	18	54	6	40
4	20	80	7	57
5	7	35	2	70
>5	*17*	*169*	*6*	*108*

$<\theta> = 0.5$, and speeds > 1000 km/s

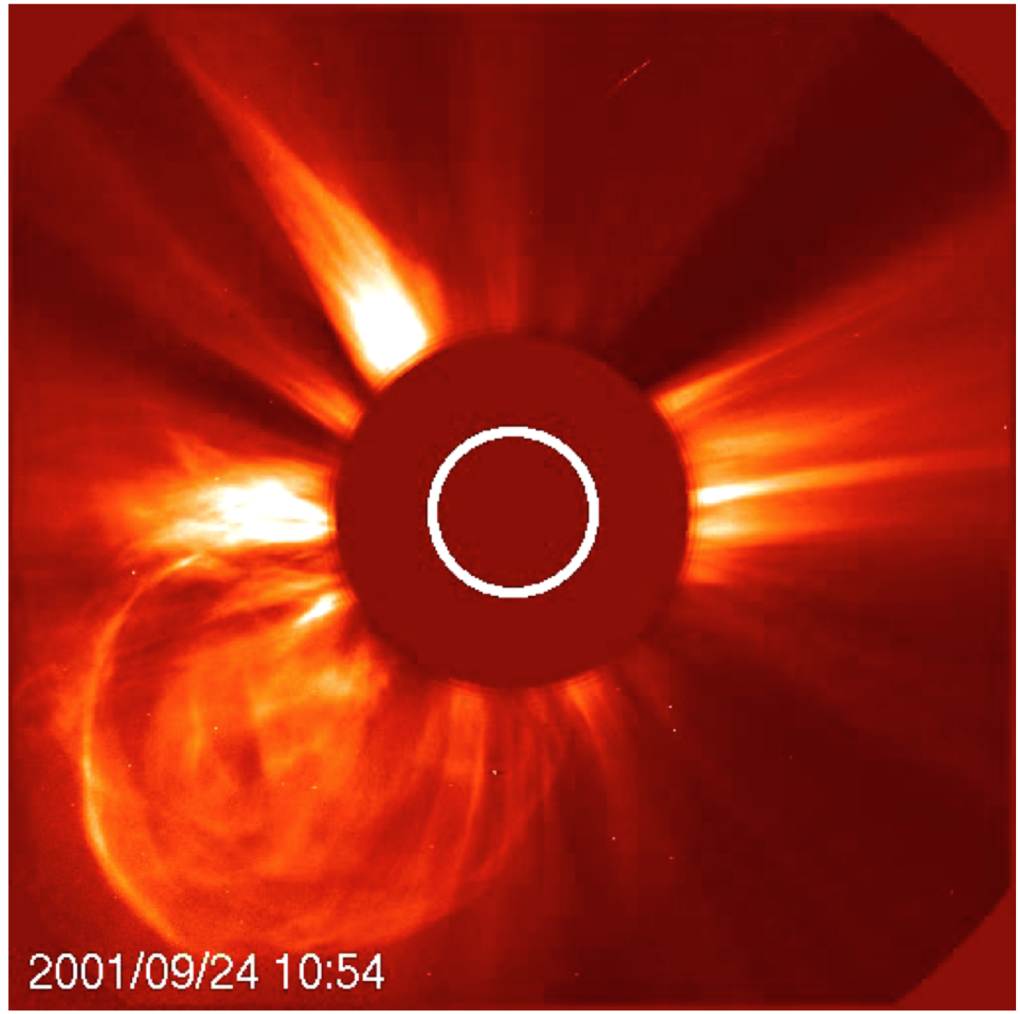
Association of Fast CME with Clusters of Active Regions

6 rotations of S. Hemisphere (180-360°)



➔

September 2001 event

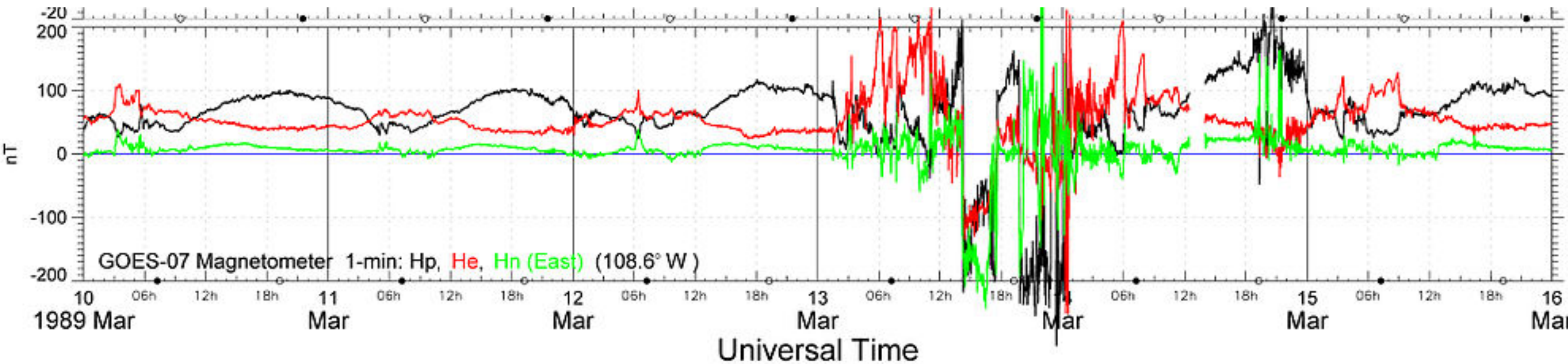


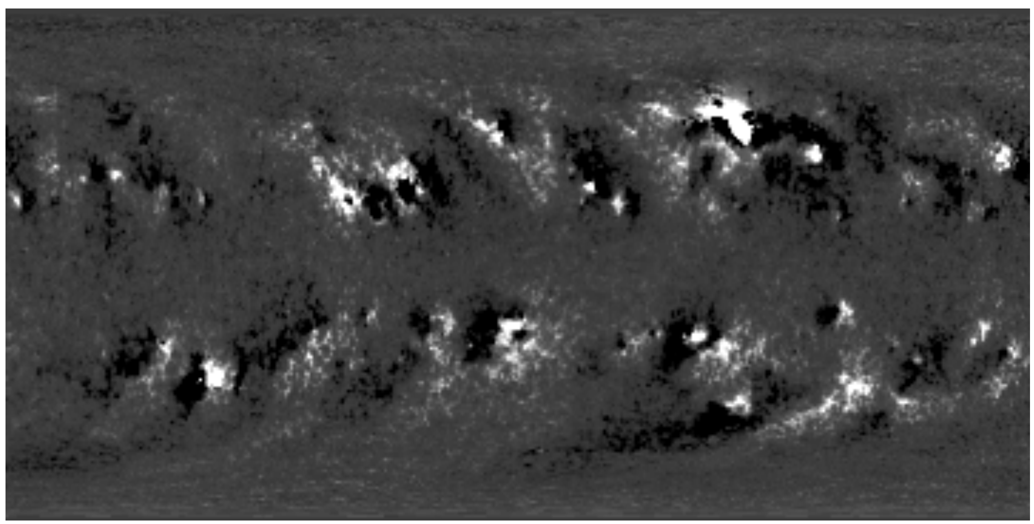
First observed at 10:31 Sep 24, 2001.
Speed 2,508 km/sec.

Halloween Event, Oct-Nov 2003

The Oct-Nov 2003 eruptions of 80 CMEs from three active regions
8 fast CMEs (V≥ 2000 km/s), (Gopalswamy,2006)

March 13, 1989, Quebec Event





On March 9 SMM observed 5 CMEs, some with speed exceeding 3000 km/s (Yakovchouk et al., ASR, 43, 2009).

Conclusions

✓ The Max Spectrum defines two scaling exponents of extreme events: α (tail exponent) and θ (extremal index, $1/\theta$ is mean number of CME in a cluster)

✓ The cumulative distribution of fast CMEs speeds asymptotically follows a power law with $\alpha \approx$ 3.2-3.7 (Fréchet extremes). This exponent defines *the distribution of high speeds, i.e. a range of fast CMEs.*

✓ The fast CMEs (and extreme SEPs associated with them) come in clusters with $<\theta> = 0.5$: If one fast CME occurs it is followed on average by one or two other fast CME in a relatively short time. The mean time between CMEs with speeds exceeding 1,000 km/s is 42 hrs.

✓There are indications that clusters of fast CMEs originate from the complex active regions (clusters of active regions).