Gravitation and the Waltz of the Planets
Chap. 4
Introduction To Modern Astronomy I: Solar System

Introducing Astronomy (chap. 1-6)

Planets and Moons (chap. 7-15)

Ch1: Astronomy and the Universe
Ch2: Knowing the Heavens
Ch3: Eclipses and the Motion of the Moon

Ch4: Gravitation and the Waltz of the Planets

Ch5: The Nature of Light
Ch6: Optics and Telescope

Chap. 16: Our Sun
Chap. 28: Search for Extraterrestrial life
Ancient Geocentric models

- Ancient astronomers believed the Earth to be at the center of the universe, and the Earth is at rest.
- All the stars are fixed on the *celestial sphere*, rotating once a day.
- The Sun and Moon move slowly eastward with respect to the stars.
Planetary Motion

- Like the Sun and Moon, the planets usually move slowly eastward on the celestial sphere with respect to the background of stars.
- This eastward progress is called **direct motion**.
- **Retrograde motion**: but from time to time, the planets stop, and move westward for several weeks or months.
Ptolemaic System: cycles on cycles

- Ptolemaic system (~ 200 AD): each planet is assumed to move in a small cycle called an epicycle, whose center in turn moves in a large cycle, called a deferent, which is centered on the Earth.

- Both the epicycle and deferent rotates in the same direction --- counter clock-wise.

![FLASH 0403003.swf](image-url)
Ptolemaic System: cycles on cycles

• When the planet is on the part of its epicycle nearest Earth, the motion of the planet along the epicycle is opposite to the motion of the epicycle along the deferent. The planet therefore appears to go backward in retrograde
Heliocentric Model by Copernicus

- **Heliocentric (Sun-centered) model**: all the planets, including the Earth, revolve about the Sun
- A heliocentric model simplifies the explanation of the retrograde motion of planets
- **Occam’s razor**: simple explanations of phenomena are most likely to be correct

Nicolaus Copernicus (1473 – 1543)
Heliocentric Model by Copernicus

- **Retrograde motion** of a planet is caused by the Earth overtaking and passing the slow-moving planet.

1. From point 1 to point 4, Mars appears to move eastward against the background of stars as seen from Earth (direct motion).
2. As Earth passes Mars in its orbit from point 4 to point 6, Mars appears to move westward against the background of stars (retrograde motion).
3. From point 6 to point 9, Mars again appears to move eastward against the background of stars as seen from Earth (direct motion).
Planetary Configurations

- **Inferior planets**: Mercury and Venus
  - Their orbits are smaller than the Earth
  - They are always observed near the Sun in the sky
- **Elongation**: the angle between the Sun and a planet as viewed from Earth

- **Greatest Eastern Elongation**
  - Mercury or Venus visible after sunset
  - Called “evening star”
- **Greatest Western Elongation**
  - Mercury or Venus visible before sunrise
  - Called “morning star”
Planetary Configurations

- **Superior planets**: Mars, Jupiter and Saturn
  - Their orbits are larger than the Earth
  - They can appear high in the sky at midnight, thus opposite the Sun with Earth in between

- **Conjunction**:  
  - The Sun and planet appear together in the celestial sphere

- **Opposition**:  
  - Earth is between Sun and planet  
  - Planet is highest in the sky at midnight  
  - Planet appears brightest because it is closest to the Earth
Synodic Period and Sidereal Period

- **Synodic period:** the time that elapses between two consecutive identical configurations as seen from the Earth
  - e.g., from one opposition to the next for superior planets
  - e.g., from one greatest eastern elongation to the next for inferior planets

- **Sidereal period:** true orbital period, the time it takes the planet to complete one full orbit of the Sun relative to the stars

- **Sidereal period** is deduced from observed *synodic period*

<table>
<thead>
<tr>
<th>table 4-1: Synodic and Sidereal Periods of the Planets</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Planet</strong></td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>Mercury</td>
</tr>
<tr>
<td>Venus</td>
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<tr>
<td>Earth</td>
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<tr>
<td>Mars</td>
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<tr>
<td>Jupiter</td>
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<tr>
<td>Saturn</td>
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<tr>
<td>Uranus</td>
</tr>
<tr>
<td>Neptune</td>
</tr>
<tr>
<td>Pluto</td>
</tr>
</tbody>
</table>
Synodic Period and Sidereal Period

- For an inferior planet, over one synoptic period

\[
\frac{1}{P} = \frac{1}{E} + \frac{1}{S}
\]

\(P\) = sidereal period of the planet
\(E\) = sidereal period of the Earth = 1 year
\(S\) = synoptic period of the planet (from observation)

For example: Mercury
\(S = 0.318 \text{ year (116 days)}\)
\(P = 0.242 \text{ year} = 88 \text{ days}\)
Synodic Period and Sidereal Period

- For an superior planet, over one synoptic period

Angular distance of the planet \((\frac{360}{P \times S}) = \)

Angular distance of the Earth \((\frac{360}{E \times S}) - 360° \)

\[
\frac{1}{P} = \frac{1}{E} - \frac{1}{S}
\]

\(P\) = sidereal period of the planet
\(E\) = sidereal period of the Earth = 1 year
\(S\) = synoptic period of the planet (from observation)

For example: Jupiter
\(S = 1.092\) year 398 days)
\(P = 11.87\) year = 4333 days
Heliocentric Model by Copernicus

- Copernicus determined the sidereal period of planets
- Copernicus also determined the distance of the planets from the Sun using trigonometry

<table>
<thead>
<tr>
<th>Planet</th>
<th>Copernican value (AU*)</th>
<th>Modern value (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>Venus</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.52</td>
<td>1.52</td>
</tr>
<tr>
<td>Jupiter</td>
<td>5.22</td>
<td>5.20</td>
</tr>
<tr>
<td>Saturn</td>
<td>9.07</td>
<td>9.55</td>
</tr>
<tr>
<td>Uranus</td>
<td>—</td>
<td>19.19</td>
</tr>
<tr>
<td>Neptune</td>
<td>—</td>
<td>30.07</td>
</tr>
<tr>
<td>Pluto</td>
<td>—</td>
<td>39.54</td>
</tr>
</tbody>
</table>

*1 AU = 1 astronomical unit = average distance from the Earth to the Sun.
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Tycho Brahe’s Observations

• Brahe’s observations measured the positions of stars and planets with unprecedented accuracy (about 1 arcmin) (before the invention of telescope)

• The data obtained by Brahe put the heliocentric model on a solid foundation.

Tycho Brahe
(1546 – 1601)
Johannes Kepler

- Using data collected by Tycho Brahe, Kepler deduced **three laws of planetary motion**, which are about

1. Orbital shape
2. Orbital speed
3. Orbital period

Johannes Kepler
(1571 – 1630)
Kepler’s First Law

• Kepler’s first law: the orbit of a planet about the Sun is an ellipse, with the Sun at one focus

• **Semimajor axis**: the average distance between the planet and the Sun

• Assuming ellipse, Kepler found his theoretical calculations match precisely to Tycho’s observations.
Ellipse

- **Eccentricity e**: the measure of the deviation from the perfect circle.
Kepler’s Second Law

- Kepler’s second law: a line joining a planet and the Sun sweeps out equal areas in equal interval of time.
- **Perihelion**: nearest the Sun; the planet moves fastest.
- **Aphelion**: farthest from the Sun; the planet moves slowest.

![Diagram](0403006.swf)
Kepler’s Third Law

- Kepler’s third law: the square of the sidereal period of a planet is directly proportional to the cube of the semimajor axis of the orbit

$$P^2 = a^3$$

$P = \text{planet’s sidereal period, in years}$

$a = \text{planet’s semimajor axis, in AU}$

<table>
<thead>
<tr>
<th>Planet</th>
<th>Sidereal period $P$ (years)</th>
<th>Semimajor axis $a$ (AU)</th>
<th>$P^2$</th>
<th>$a^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>0.24</td>
<td>0.39</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Venus</td>
<td>0.61</td>
<td>0.72</td>
<td>0.37</td>
<td>0.37</td>
</tr>
<tr>
<td>Earth</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Mars</td>
<td>1.88</td>
<td>1.52</td>
<td>3.53</td>
<td>3.51</td>
</tr>
<tr>
<td>Jupiter</td>
<td>11.86</td>
<td>5.20</td>
<td>140.7</td>
<td>140.6</td>
</tr>
<tr>
<td>Saturn</td>
<td>29.46</td>
<td>9.55</td>
<td>867.9</td>
<td>871.0</td>
</tr>
<tr>
<td>Uranus</td>
<td>84.10</td>
<td>19.19</td>
<td>7,072</td>
<td>7,067</td>
</tr>
<tr>
<td>Neptune</td>
<td>164.86</td>
<td>30.07</td>
<td>27,180</td>
<td>27,190</td>
</tr>
<tr>
<td>Pluto</td>
<td>248.60</td>
<td>39.54</td>
<td>61,800</td>
<td>61,820</td>
</tr>
</tbody>
</table>
Kepler’s Laws

- Kepler’s laws of planetary motion are a landmark in astronomy
- They made it possible to calculate the motions of planets with better accuracy than any geocentric model ever had
- They passed the test of Occam’s razor
- They helped to justify the idea of heliocentric models
Galileo’s Discoveries with Telescope

• The invention of the telescope in the early 17th century changed astronomy forever.
• Galileo’s discoveries forever changed people’s view about heavens.

Galileo Galilei
(1564 – 1642)
Galileo’s Discoveries

- Galileo discovered four moons, now called the Galilean satellites, orbiting Jupiter
  - Io, Europa, Ganymede and Callisto
- The Earth is not at the center of all heavenly objects.
- He also discovered
  - The Milky Way is not a featureless band of light, but “a mass of innumerable stars”
  - Mountains on the Moon
  - Sunspot on the Sun
  - Ring of Saturn
Phases of Venus

- Venus exhibits phases like those of the Moon
- The apparent size ($\alpha$) is related to the planet’s phase
  - Venus appears larger at crescent phase
  - Venus appears smaller at gibbous phase

$\alpha$: apparent angular size of Venus as seen through telescope.

Correction: the unit should be ” (arcsec) instead of ° (degree)
• Heliocentric model provides a natural explanation for the phases of Venus
  – When Venus is on the same side of the Sun as the Earth, we see it a “new” phase and with a larger angular size
  – When Venus is on the opposite side of the Sun from the Earth, it appears full and has a small angular size
Phases of Venus

- Ptolemaic geocentric model was wrong

- To explain why Venus is never seen very far from the Sun, the Ptolemaic model had to assume that the deferents of Venus and of the Sun move together in lockstep, with the epicycle of Venus centered on a straight line between the Earth and the Sun.

- In this model, Venus was never on the opposite side of the Sun from the Earth, and so it could never have shown the gibbous phases that Galileo observed.
Isaac Newton

Isaac Newton, based on the insight into fundamental principles, introduced

- three laws of motion
  - Law of Inertia
  - Law of Force
  - Law of Action and Reaction

- the law of universal gravitation

Isaac Newton (1642 -- 1727)
Newton First Law of Motion

- First law of motion, or law of inertia:
  A body remains at rest, or moves in a straight line at a constant velocity, unless acted upon by a net outside force

- **Speed**: a measure of how fast an object is moving
- **Velocity**: the combination of speed and direction of motion
Newton Second Law of Motion

- Second law of motion, or law of force:
  The acceleration of an object is proportional to the net outside force acting on the object, and is inversely proportional to the mass of the object.
  \[ F = ma \]
  
  \( F \) = net outside force on an object
  \( m \) = mass of object
  \( a \) = acceleration of object

- **Mass**: total amount of material in the object, an intrinsic value independent of gravitational environment; measured in Kg (Kilogram)
- **Acceleration**: the rate at which velocity changes
- **Weight**: force of gravity that acts on a body; measured in Newton (1 Newton = 0.225 Pound)
- Earth surface gravity = 9.8 m/s\(^2\)
- Mars surface gravity = 3.7 m/s\(^2\) (0.4 \(g_E\))
Newton Third Law of Motion

- Third law of motion, or law of action and reaction: *Whenever one body exerts a force on a second body, the second body exerts an equal and opposite force on the first body*
Newton’s Law of Universal Gravitation

- Law of Universal Gravitation:
  Two bodies attract each other with a force that is directly proportional to the mass of each body and inversely proportional to the square of the distance between them

\[ F = G \left( \frac{m_1 m_2}{r^2} \right) \]

- \( F \) = gravitational force between two objects
- \( m_1 \) = mass of first object
- \( m_2 \) = mass of second object
- \( r \) = distance between objects
- \( G \) = universal constant of gravitation: \( 6.67 \times 10^{-11} \) newton \( \cdot \) m\(^2\)/kg\(^2\)
Gravitation: Orbital Motions

- Kepler’s three laws of planetary motion can be exactly derived from Newton’s law of universal gravitation

- E.g.,
  - closer to the Sun
  - stronger the gravitational force
  - faster the orbital speed
  - smaller the orbital period
Gravitation: Orbital Motions

- Why not Earth falling into the Sun nor the Moon crashing into the Earth?
- Paths A, B, and C do not have enough horizontal velocity to escape Earth’s surface whereas Paths D, E, and F do.
- Path E is where the horizontal velocity is exactly what is needed so its orbital curve matches the circular curve of the Earth.

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Gravitation: Orbital Motions

- Based on his gravitational law, Newton found that the orbits of an object around the Sun could be any one of a family of curves called conic sections.
- Some comets are found to have hyperbolic orbits.
Gravitation: Tidal Force

- Tidal forces are differences in the gravitational pull at different points in an object.

- From the perspective of the center ball, it appears that the forces have pushed the 1-ball away and pulled the 3-ball toward the planets.
Tidal Force

• The tidal force equals the Moon’s gravitational pull at the location minus the gravitational pull of the Moon at the center of the Earth.

• These tidal forces tend to deform the Earth into a non-spherical shape.

From the perspective of the center of the Earth
Tidal Force

- The positions of high tide caused by the Moon
  - Moon is at the upper local meridian (highest in the sky)
  - Moon is at the lower local meridian
Tidal Force

- **Spring tide**
  - the highest tide, when the tidal effects of the Sun and Moon reinforce each other
  - Happens at either new moon or full moon

- **Neap tide**
  - the smallest tide, when the tidal effects of the Sun and Moon partially cancelled each other
  - Happens at either first quarter or third quarter
Final Notes on Chap. 4

• There are 8 sections. All the sections are covered.
• There are 4 boxes. All boxes are covered.
Advanced Question
Chap. 4, Q43 in P93

Suppose that you travelled to a planet with 4 times the mass and 4 times the diameter of the Earth. Would you weigh more or less on that planet than on Earth? By what factor?