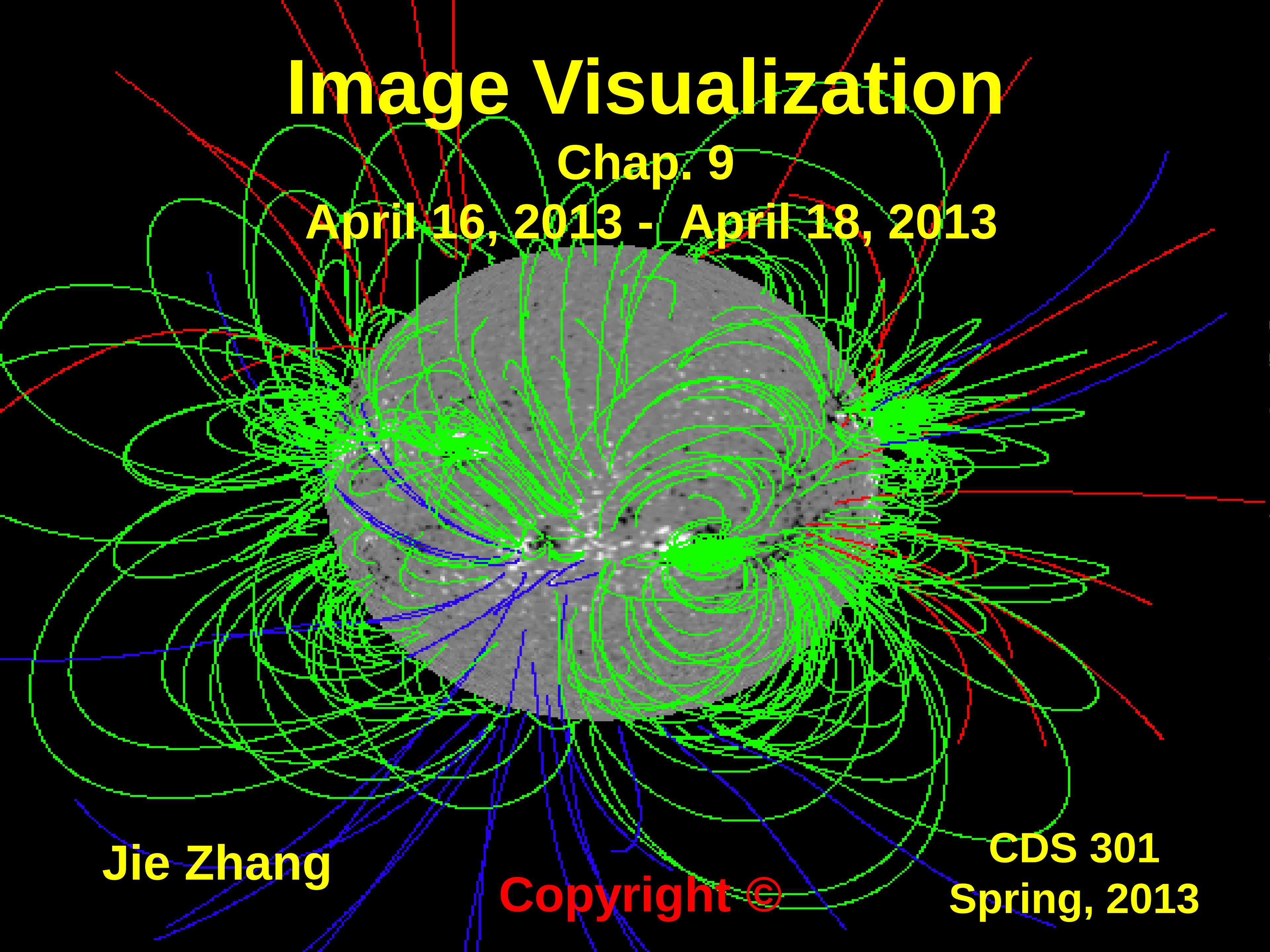
Image Visualization

Chap. 9

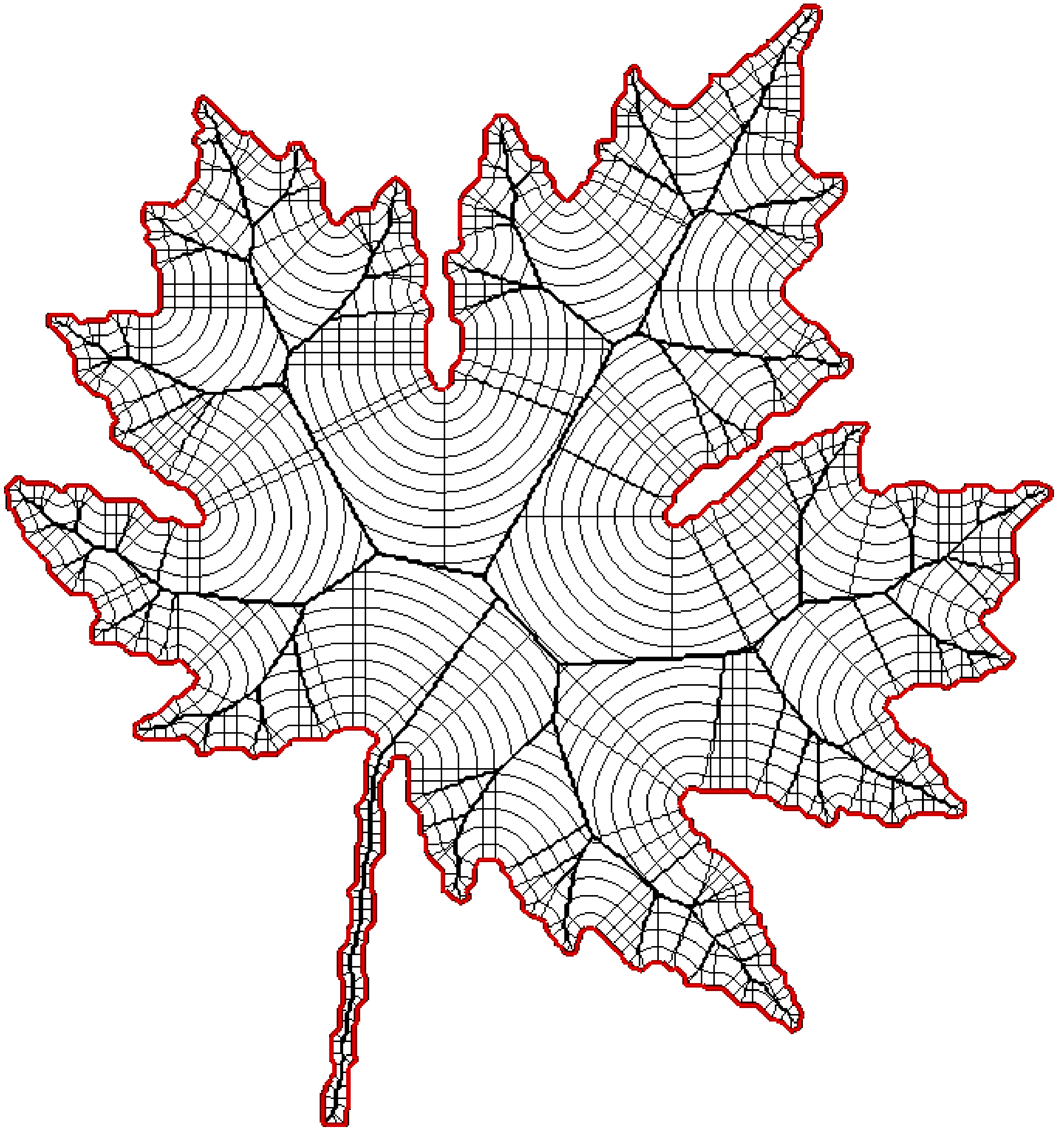
April 16, 2013 - April 18, 2013



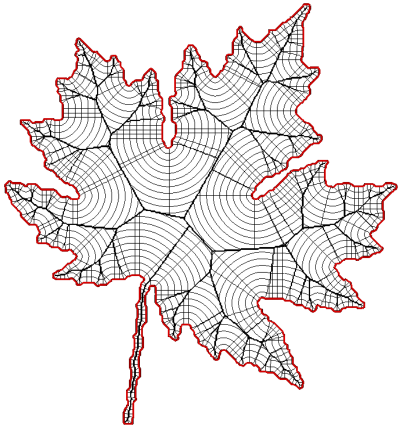
Jie Zhang

CDS 301
Spring, 2013

Image Visualization



Outline



9.1. Image Data Representation

9.2. Image Processing and Visualization

9.3. Basic Imaging Algorithms

- Contrast Enhancement
- Histogram Equalization
- Gaussian Smoothing
- Edge Detection

9.4. Shape Representation and Analysis

- Segmentation
- Connected Components
- Morphological Operations
- Distance Transforms
- Skeletonization

CH9.1 Image Data Representation

April 16, 2013

Image Data Representation

What is an image?

- An image is a well-behaved uniform dataset.
- An image is a two-dimensional array, or matrix of pixels, e.g., bitmaps, pixmaps, RGB images
- A pixel is square-shaped
- A pixel has a constant value over the entire pixel surface
- The value is typically encoded in 8 bits integer

$$\boldsymbol{D_s} = (\{p_i\}, \{C_i\}, \{f_i\}, \{\Phi_i\})$$

CH9.2 Image Processing and Visualization

April 16, 2013

Image Processing and Visualization

- Image processing follows the visualization pipeline
  - At the end of the Visualization Pipeline, the rendering always produces a 2-D image
  - The 2-D image can be further processed using image processing methods, such as contrast enhancement and color adjustment.

CH9.3 Basic Imaging Algorithms

April 16, 2013

CH9.3.1 Basic Image Processing

- Image enhancement operation is to apply a transfer function on the pixel luminance values

- Transfer function is usually based on image histogram analysis

- High-slope function enhance image contrast
- Low-slope function attenuate the contrast.

Basic Image Processing

- The basic image processing is the contrast enhancement through applying a transfer function

Transfer function :

(1) The original image :

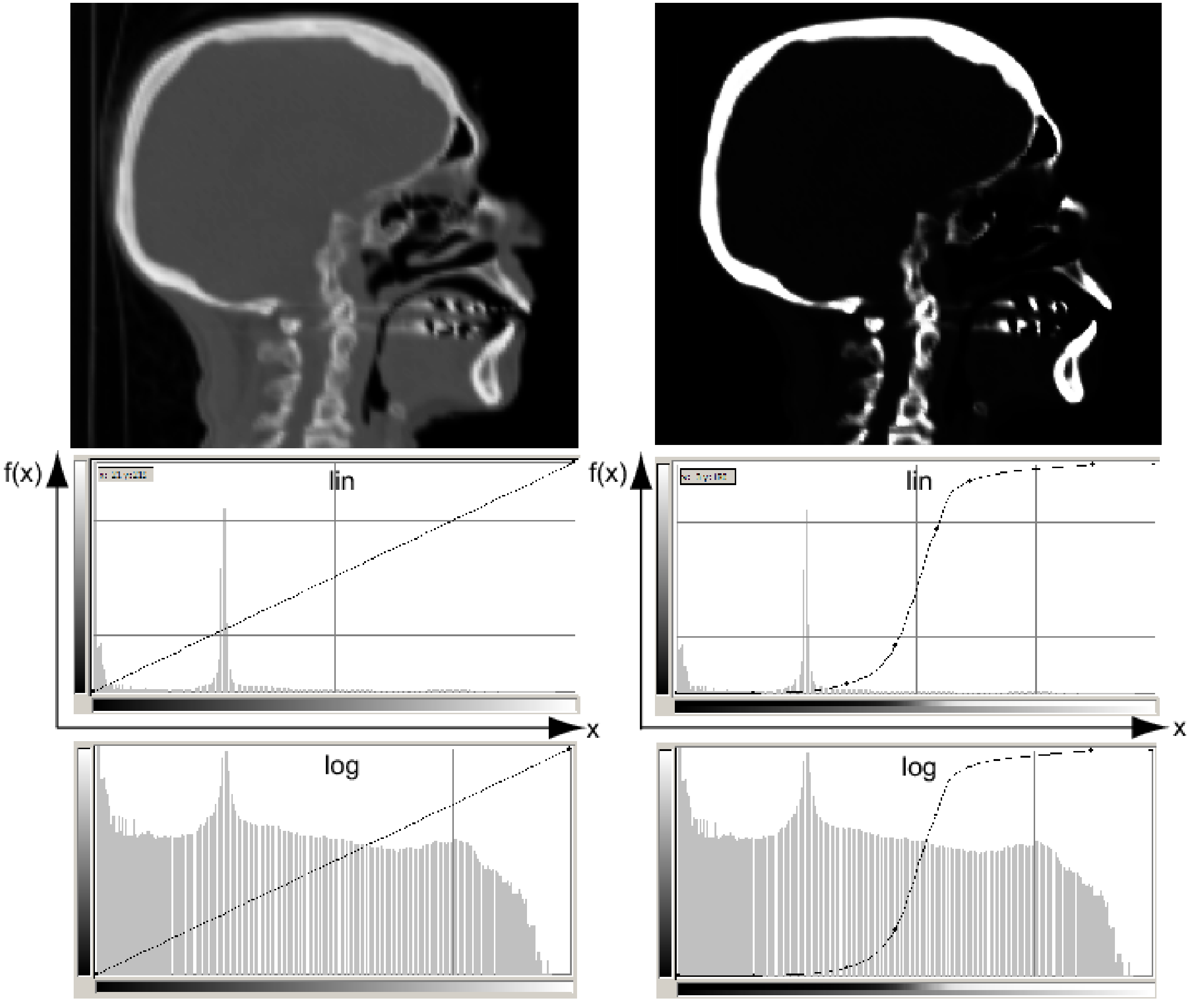
$f(x) = x$

(2) Linear normalization

$f(x) = (x - I_{min})/(I_{max} - I_{min})$

(3) Non-linear Transfer

Image Enhancement



a) original

b) contrast-enhanced

Linear Transfer

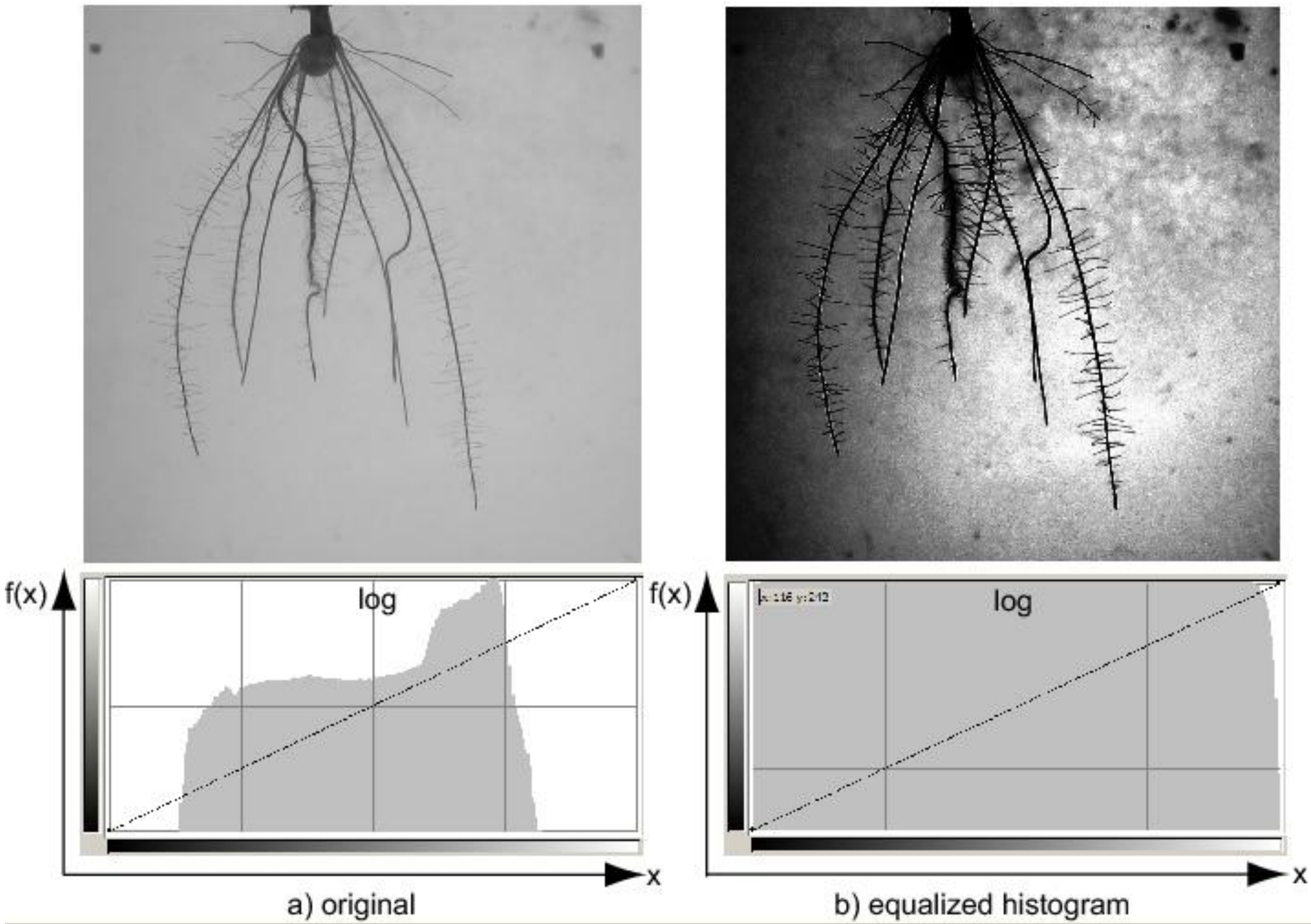
Non-linear Transfer

CH9.3.2 Histogram Equalization

- Histogram equalization method is to compute a transfer function such as the resulted image has a near-constant histogram
- Original high density regions (peaks in original histogram) get more colors

$$f(x) = (size - 1) \sum_{i=0}^{x} h[i]$$

Histogram Equalization



Original Image

After equalization

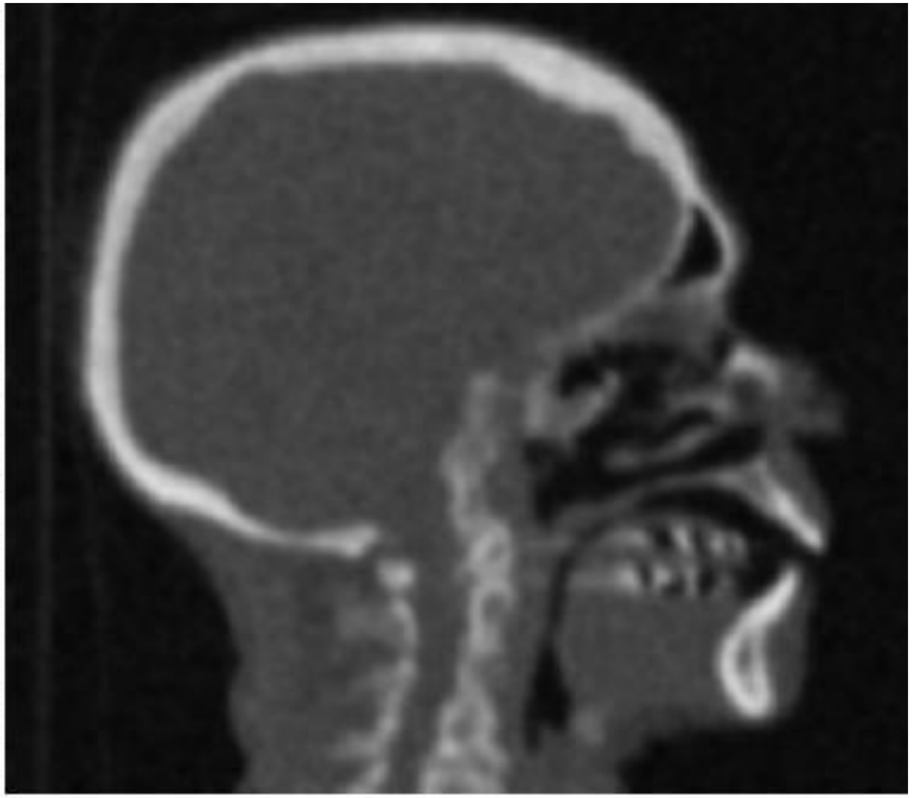
CH9.3.3 Gaussian Smoothing

How to remove noise?

- Noise can be described as rapid variation of high amplitude
- Or regions where high-order derivatives of f have large values
- Noise is usually the high frequency components in the Fourier series expansion of the input signal

Smoothing

a)



b)

Noise image

After filtering

Fourier Series

- For any continuous function f(x) with period T (or x=[0,T], the boundary of the data), the Fourier series expansion are:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin(w_n x) + \sum_{n=1}^{\infty} b_n \cos(w_n x)$$

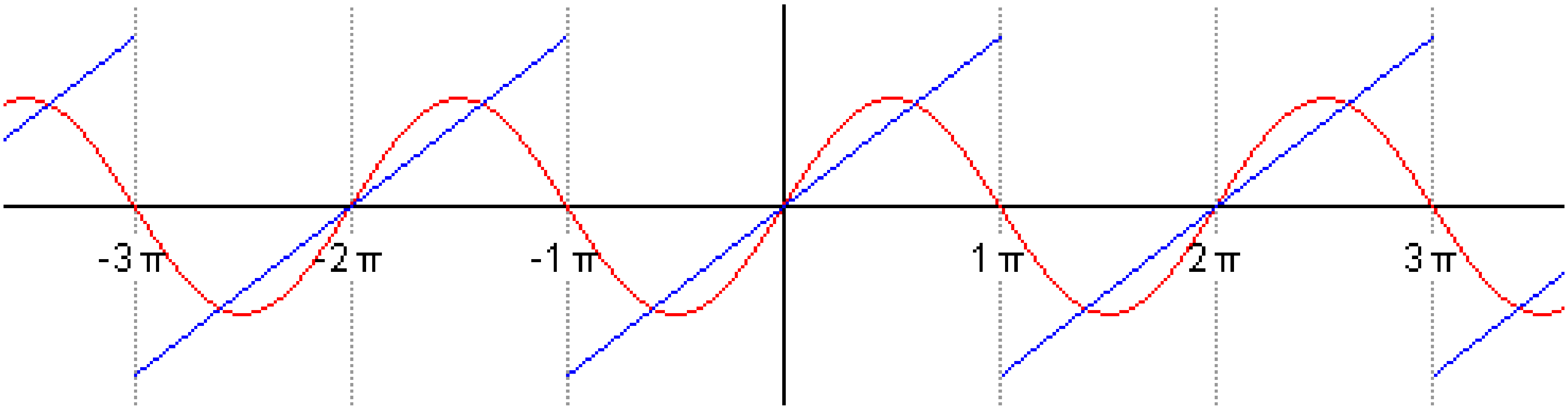
$$w_n = n\frac{2\pi}{T}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\sin(w_n t)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\cos(w_n t)dt$$

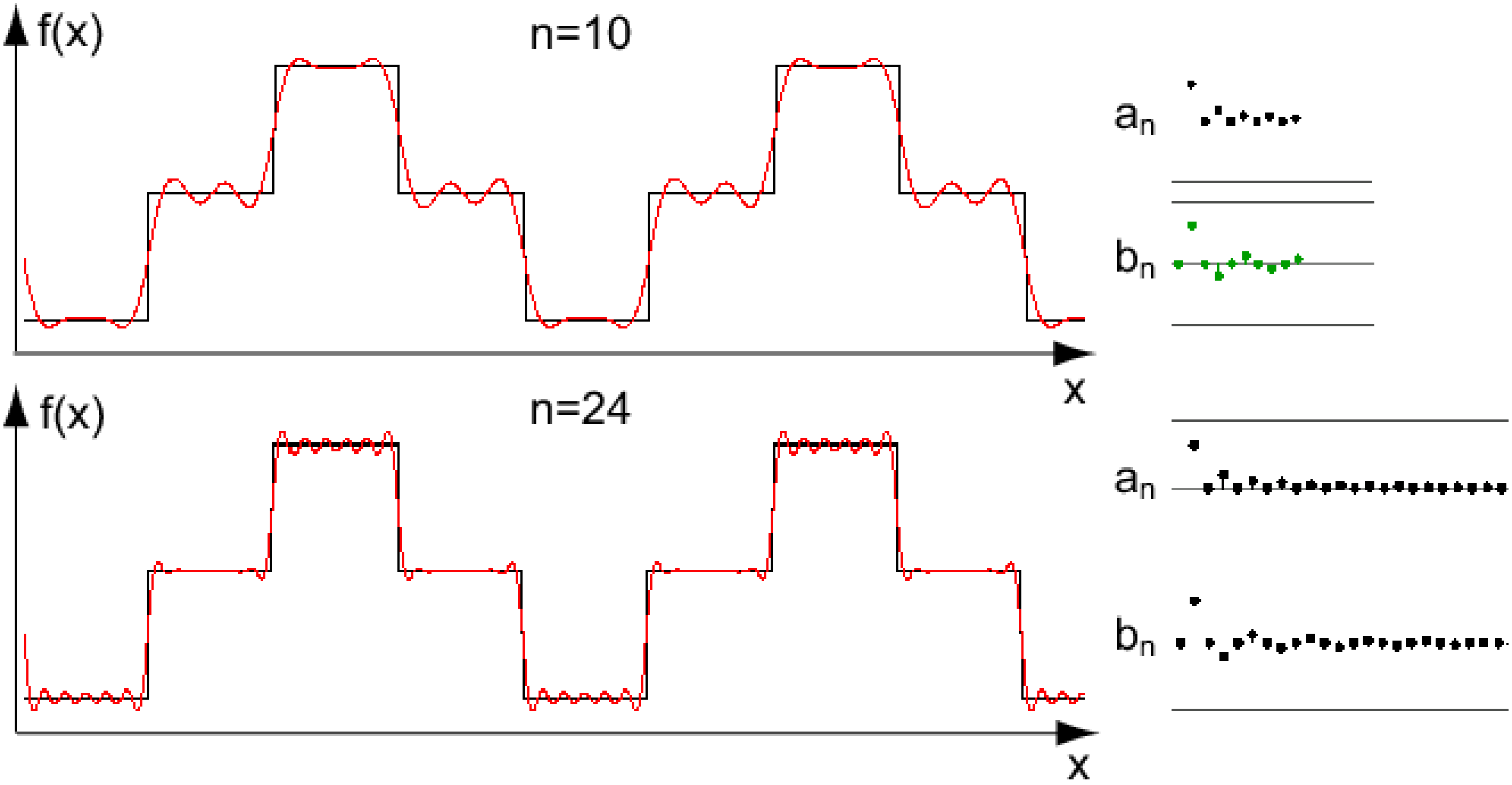
The higher the order n or the frequency, the smaller the amplitudes a_n and b_n

Fourier Series



http://en.wikipedia.org/wiki/Fourier_series

Fourier Series



Fourier Transform

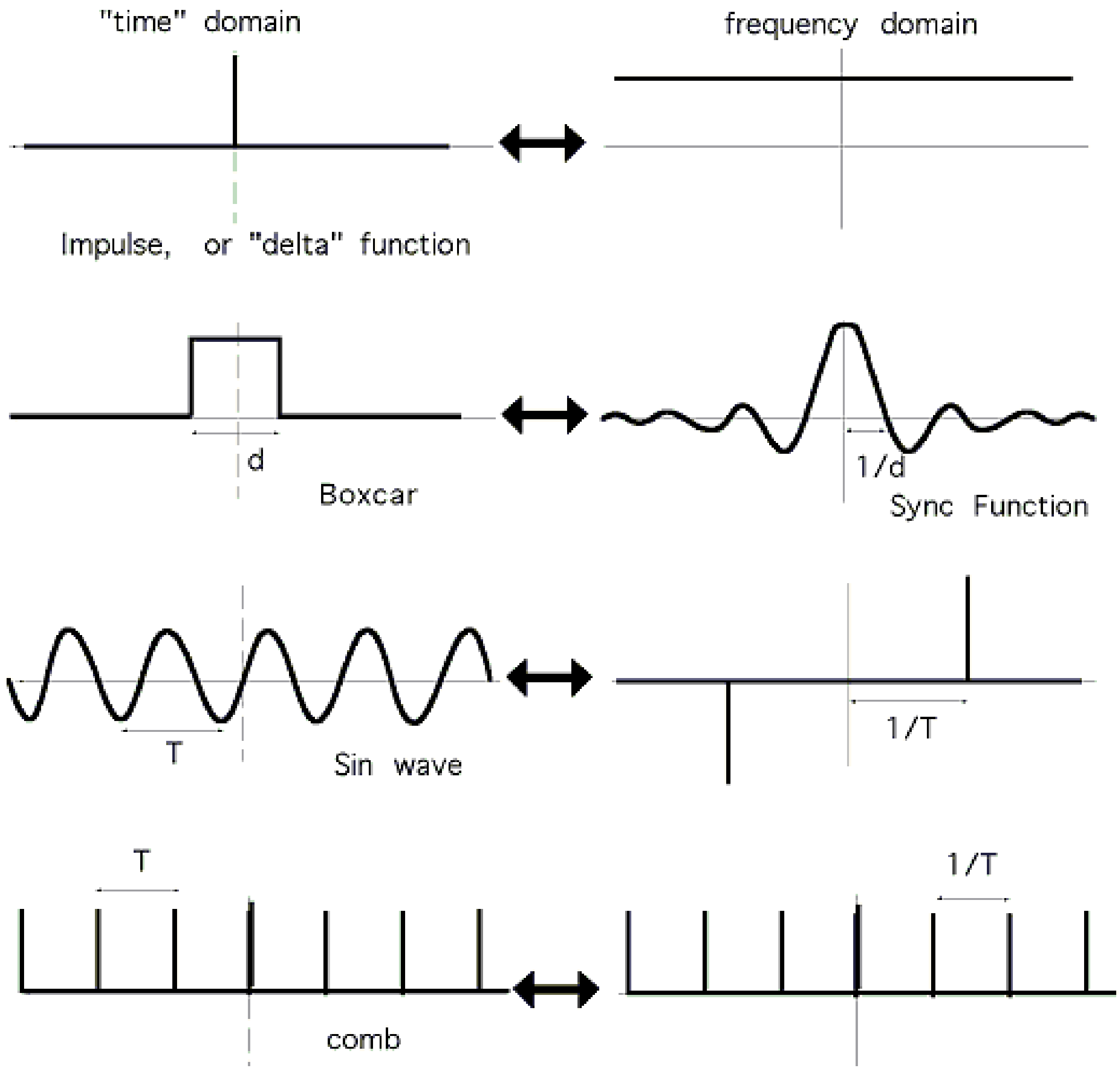
When $\mathrm{T} \to \infty$, w is continuous, amplitudes are also continuous.

$$A(w) = \int_0^\infty f(t)\sin(wt)dt$$

$$B(w) = \int_0^\infty f(t)\cos(wt)dtB$$

$$F(w) = (A(w), B(w))$$

Fourier Transform



Frequency Filtering

1. Computer the Fourier transform $F(w_x,w_y)$ of $f(x,y)$

2. Multiple F by the filtering function Φ to obtain a new function G, e.g., high frequency components are removed or attenuated.

3. Compute the inverse Fourier transform G^{-1} to get the filtered version of f

$$f \rightarrow F$$

$$G = F \cdot \Phi$$

$$f = G^{-1}$$

Frequency Filtering

Frequency filter function Φ can be classified into three different types:

1. Low-pass filter: increasingly damp frequencies above some maximum w_{max}
2. High-pass filter: increasingly damp frequencies below some minimal w_{min}
3. Band-pass filter: damp frequencies with some band $[w_{min}, w_{max}]$

To remove noise, low-pass filter is used

Gaussian smoothing

The most-used low-pass filter is the Gaussian function

$$F(e^{-ax^2}) = \sqrt{\frac{\pi}{a}} e^{-\pi^2 \omega^2 / a}$$

Convolution Theorem

$$(f(x) * g(x)) = \int_{-\infty}^{\infty} f(t) g(x-t) dt$$

$$(f(x) * g(x)) \leftrightarrow F \cdot G$$

$$(f * g)_i = \sum_{k=0}^{N} f_k \, g_{N+i-k}$$

Frequency filtering is equivalent to the convolution with a filter function g(x)

April 18, 2013

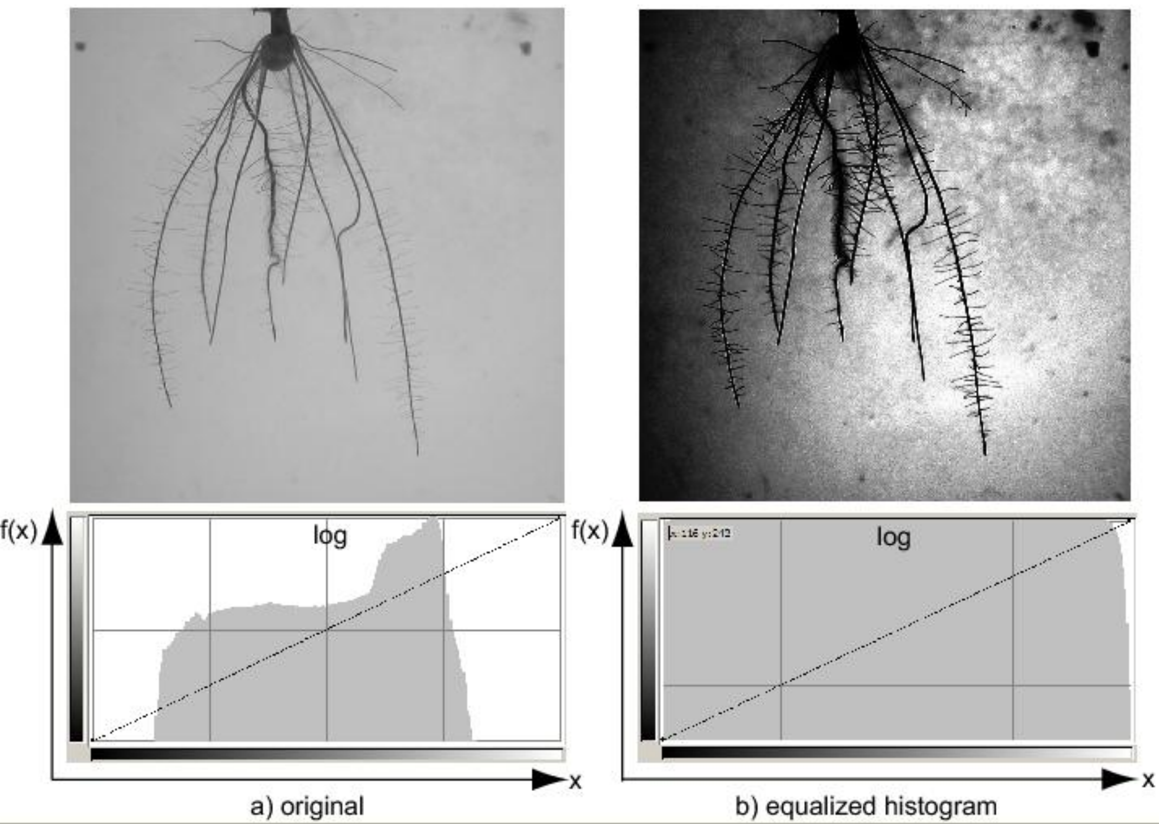
Review: Image Processing

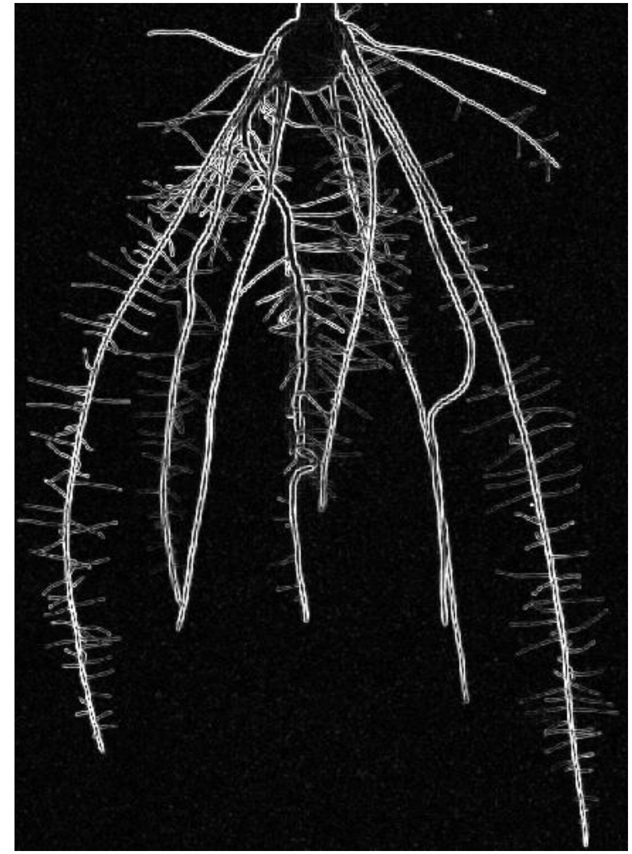
- Image enhancement operation is to apply a transfer function on the pixel luminance values

- Transfer function is usually based on image histogram analysis
  - Non-linear transfer: highlight
  - Histogram equalization: bring out the detail

- Low-pass filter: reduce noise

CH9.3.4 Edge Detection





Original Image

Edge Detection

Edge Detection

- Edges are curves that separate image regions of different luminance
- Edges are locations that have high gradient

$$\left|\nabla I(x,y)\right| = \sqrt{(\frac{\partial I}{\partial x})^2 + (\frac{\partial I}{\partial y})^2}$$

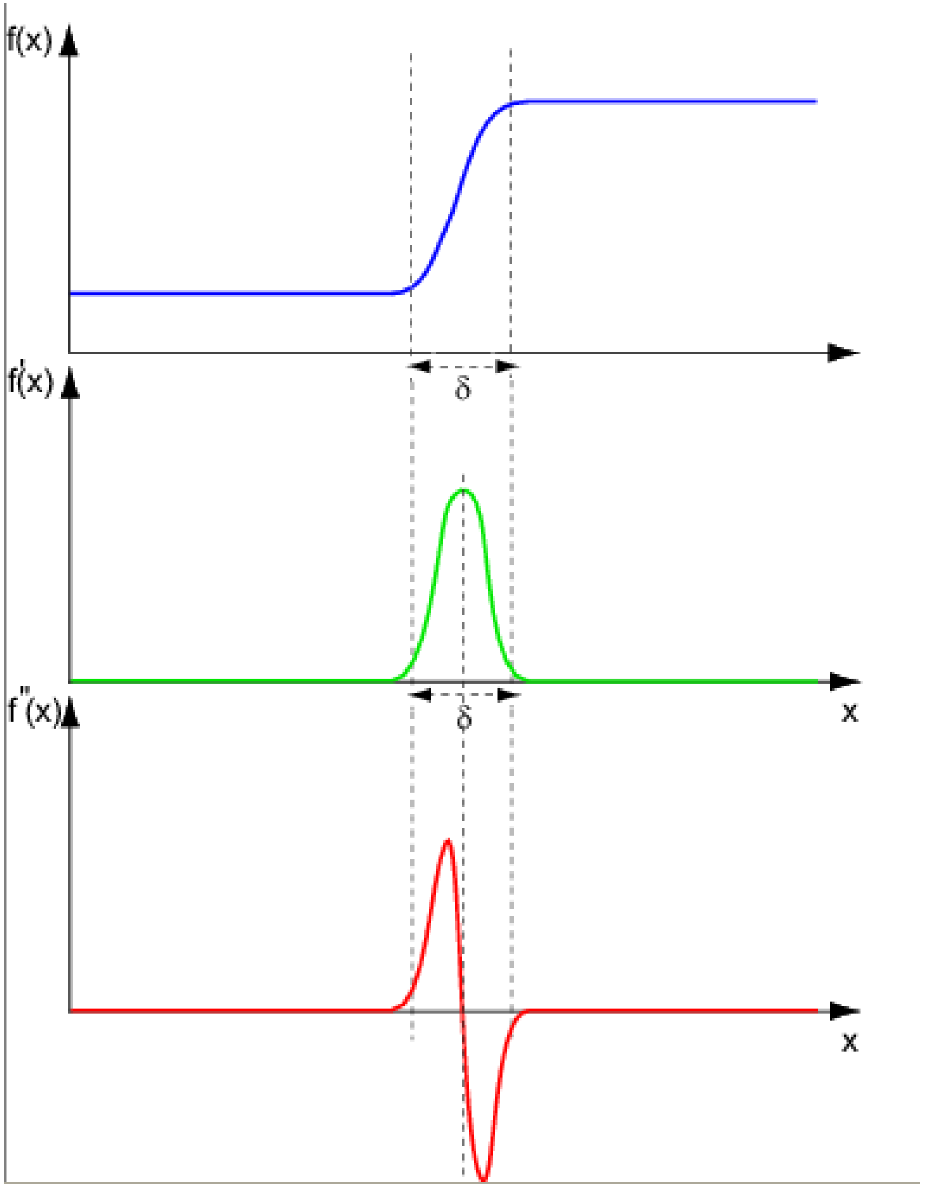
$$\frac{\partial I}{\partial x}(i, j) = I_{i+1, j} - I_{i, j}$$

$$\frac{\partial I}{\partial y}(i, j) = I_{i, j+1} - I_{i, j}$$

Edge Detection

Edges detection using derivatives for continuous function:

(1) maximum of the first derivative;
(2) zero-crossing of the 2nd derivative



Edge Detection Operators

$$R(i,j) = \sqrt{(I_{i+1,j+1} - I_{i,j})^2 + (I_{i+1,j} - I_{i,j+1})^2}$$

Roberts Operator

$$\frac{\partial I}{\partial x}(i,j) = I_{i+1,j-1} + 2I_{i+1,j} + I_{i+1,j+1} - I_{i-1,j-1} - 2I_{i-1,j} - I_{i-1,j+1}$$

$$\frac{\partial I}{\partial y}(i,j) = I_{i+1,j+1} + 2I_{i,j+1} + I_{i-1,j+1} - I_{i+1,j-1} - 2I_{i,j-1} - I_{i-1,j-1}$$

Sobel Operator: good on noise

These are the first-order derivative. Finding edge is to find the high value of gradient through thresholding segmentation

Edge Detection Operators

Laplacian-based operator:
good on producing thin edge

Second-order derivative. Finding edge is to find the zero-crossing or minimum.

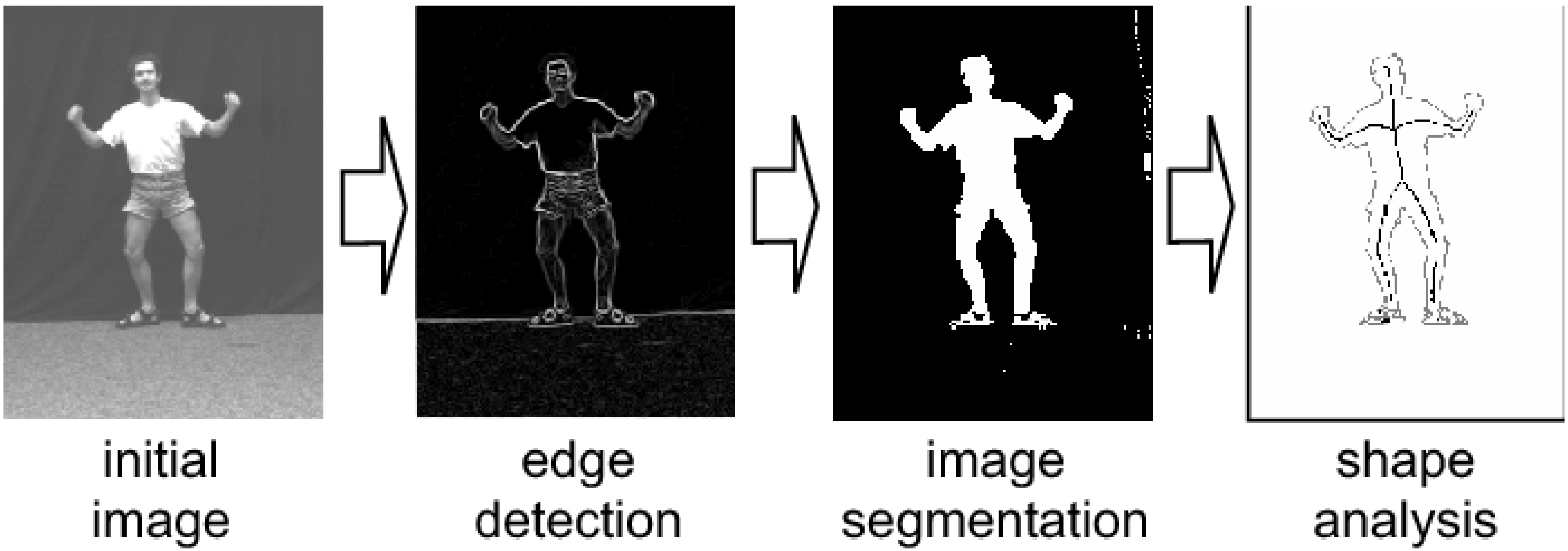
$$\left|\Delta I(x,y)\right| = \left|\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}\right|$$

$$\Delta I(i,j) = 4I_{i,j} - I_{i+1,j} - I_{i-1,j} - I_{i,j+1} - I_{i,j-1}$$

CH9.4 Shape Representation and Analysis

April 18, 2013

Shape Representation and Analysis



Shape Analysis Pipeline

Shape Representation and Analysis

- Filtering high-volume, low level datasets into low volume dataset containing high amounts of information
- Shape is defined as a compact subset of a given image
- Shape is characterized by a boundary and an interior
- Shape properties include
  - geometry (form, aspect ratio, roundness, or squareness)
  - Topology (genus number)

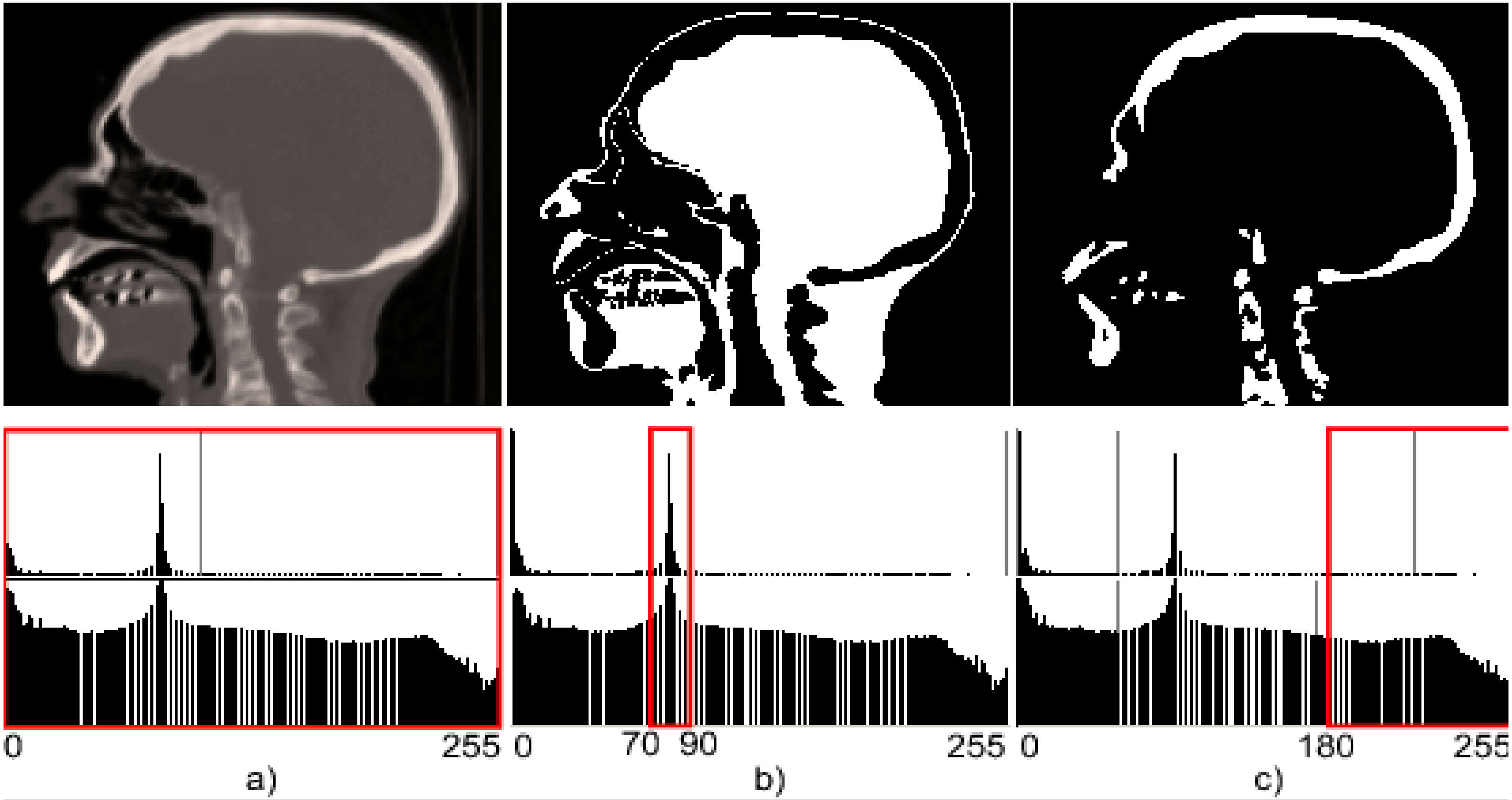
CH9.4.1 Segmentation

- Segment or classify the image pixels into those belonging to the shape of interest, called foreground pixels, and the remainder, called background pixels.

- Segmentation results in a binary image

- Segmentation is related to the operation of selection, i.e., thresholding

Segmentation



Find soft tissue

Find hard tissue

CH9.4.2 Connected Components

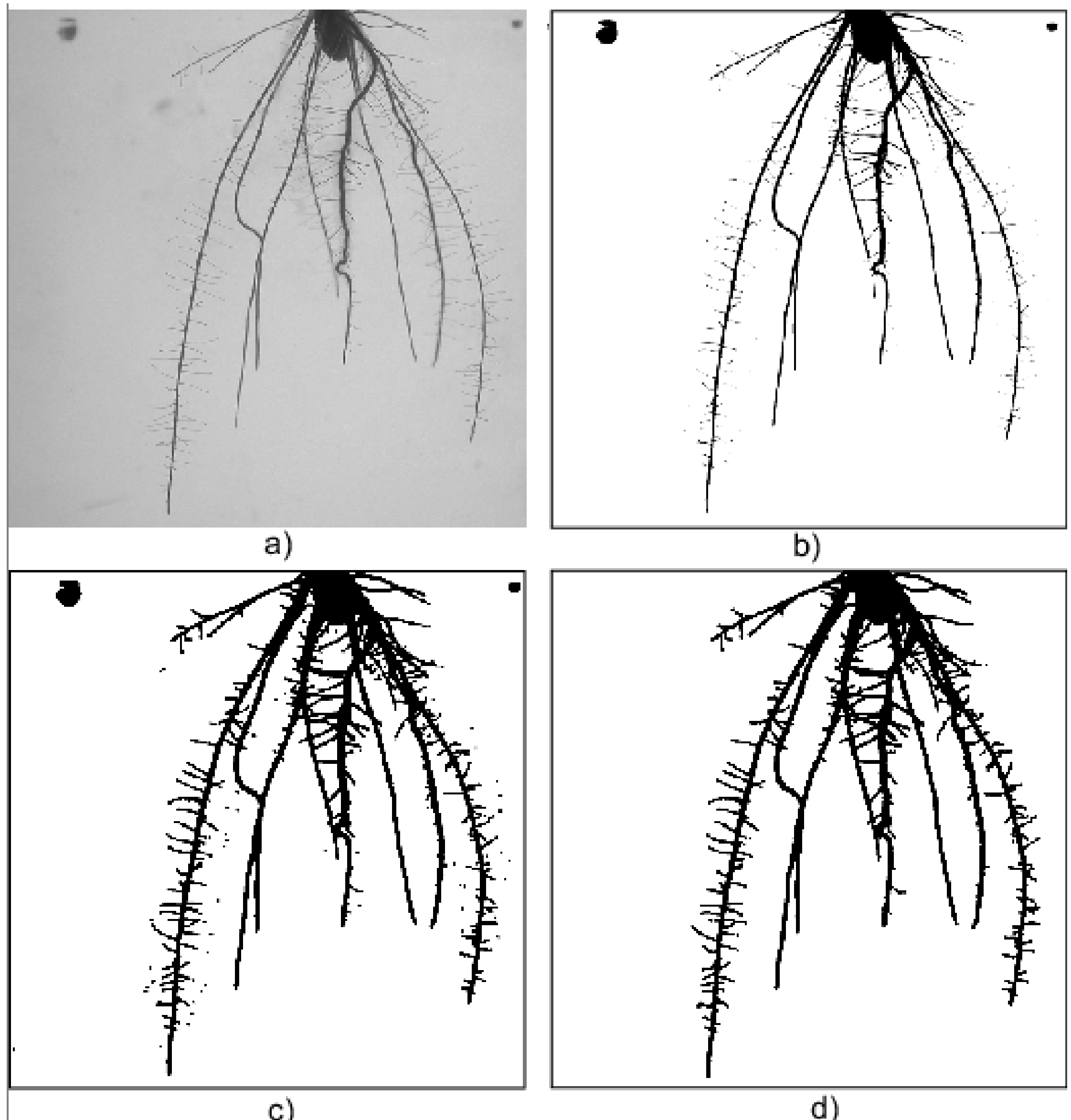
Find non-local properties

Algorithm: start from a given foreground pixel, find all foreground pixels that are directly or indirectly neighbored

CH9.4.3 Morphological Operations

To close holes and remove islands in segmented images

a: original image
b: segmentation
c: close holes
d: remove island



Morphological Operations

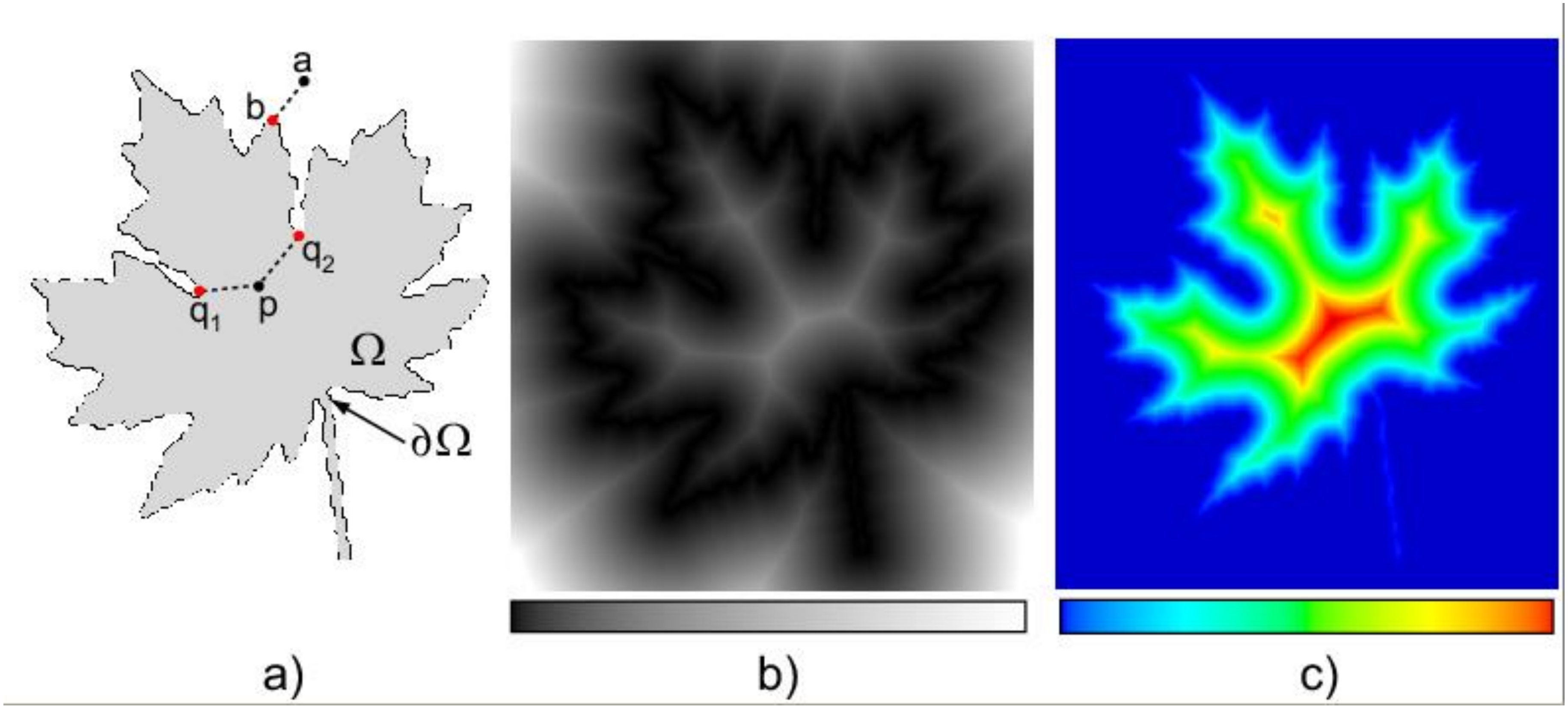
- **Dilation**: translate a structuring element (e.g., disc, square) over each foreground pixel of the segmented image
- Dilation thickens thin foreground regions, and fill holes and close background gaps that have a size smaller than the structuring element R

- **Erosion**: the opposite operation of dilation.
- Erosion is to thin the foreground components, remove island smaller than the structuring element R

Morphological Operations

- Morphological closing: dilation followed by an erosion --- close small holes
- Morphological opening: erosion followed by a dilation operation --- remove small islands

CH9.4.4 Distance Transform



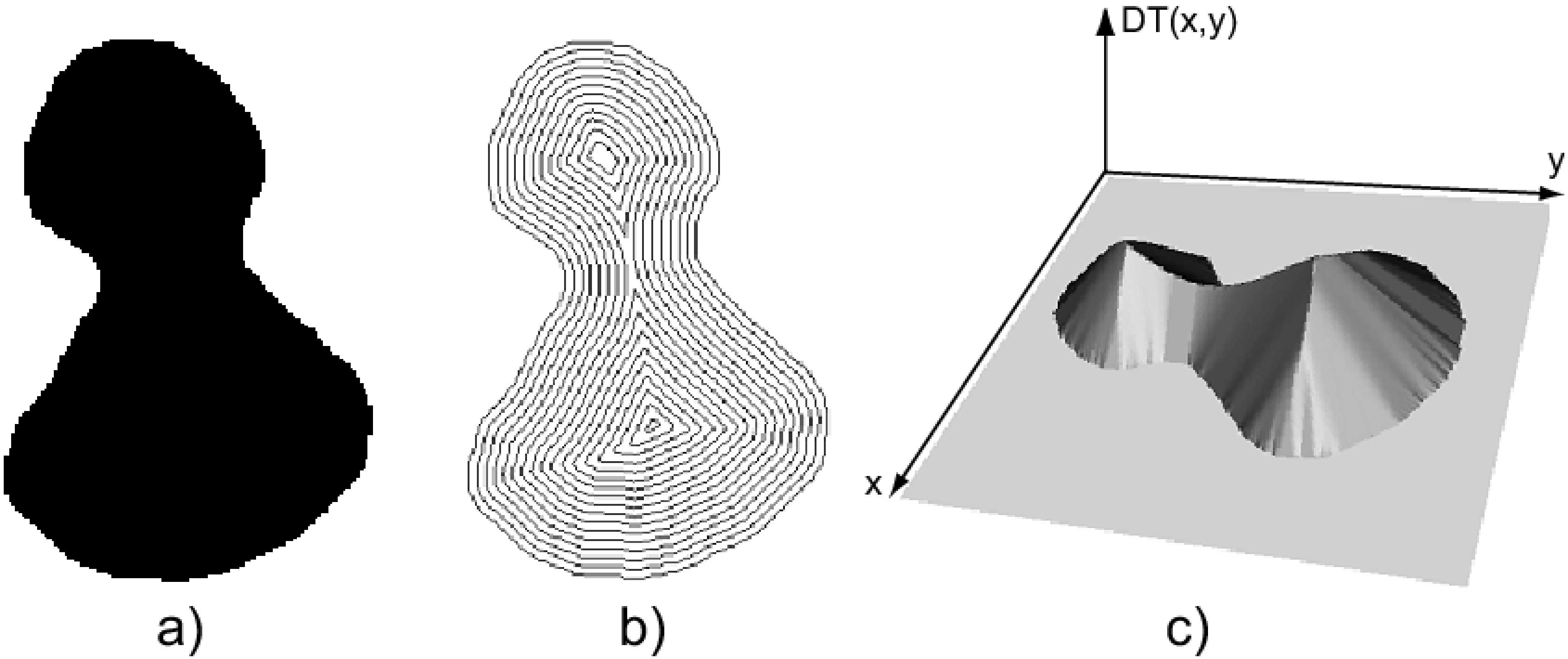
Distance Transform

- The distance transform DT of a binary image I is a scalar field that contains, at every pixel of I, the minimal distance to the boundary ∂ Ω of the foreground of I

$$DT(p) = \min_{q \in \partial\Omega} |p - q|$$

Distance Transform

- The contour lines of DT are also called level sets



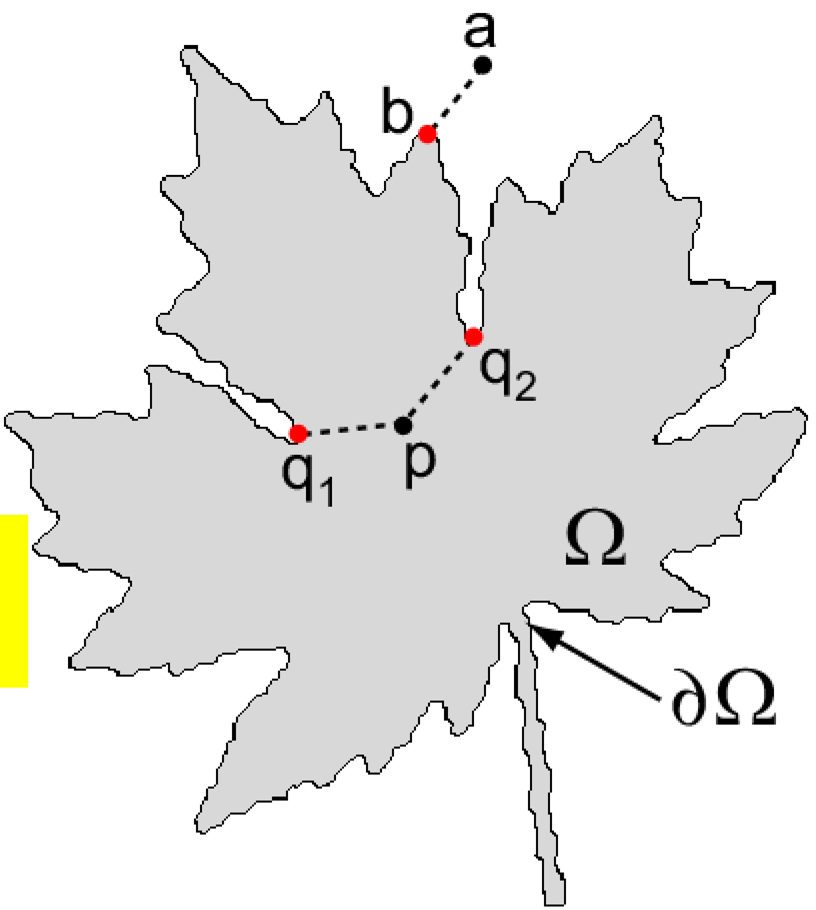
Shape **Level Sets** **Elevation plot**

Feature Transform

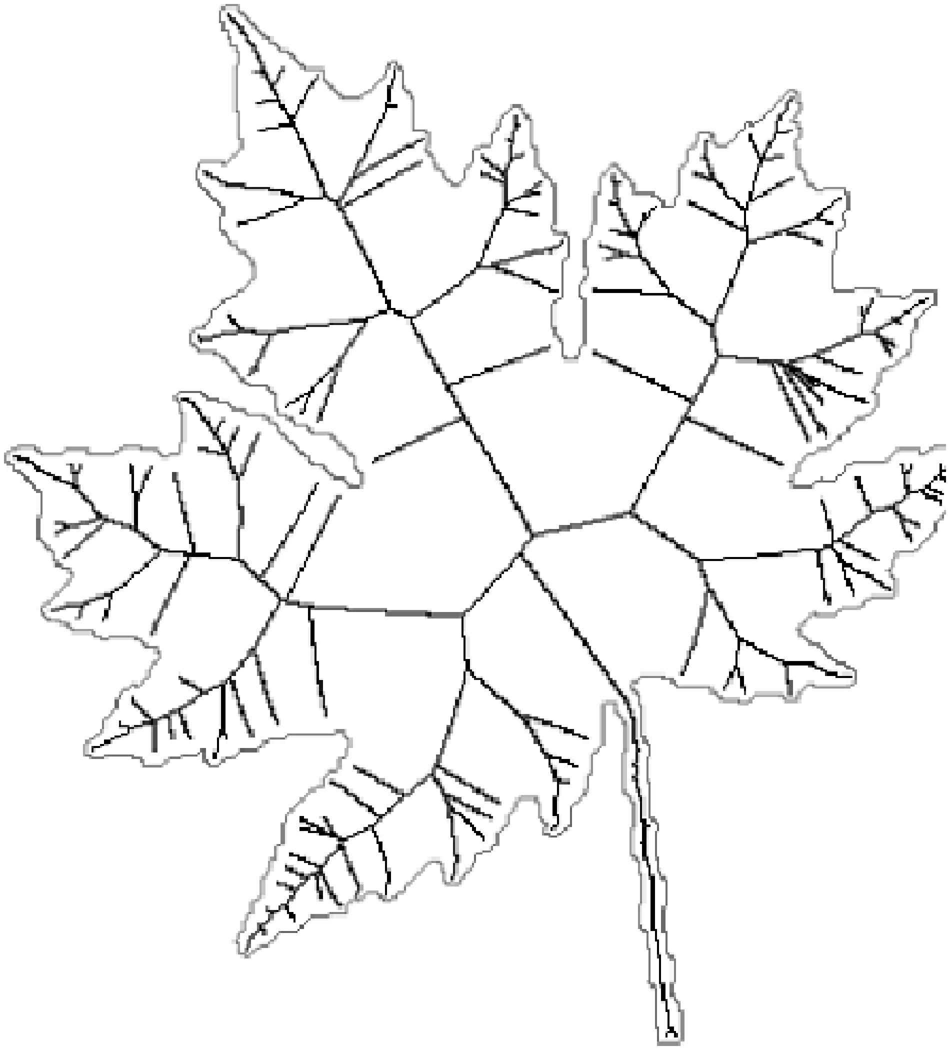
- Find the closest boundary points, so called feature points

Given a:
Feature point is b

Given p:
Feature points are q_1 and q_2



CH9.4.5 Skeletonization



Skeletonization: the Goals

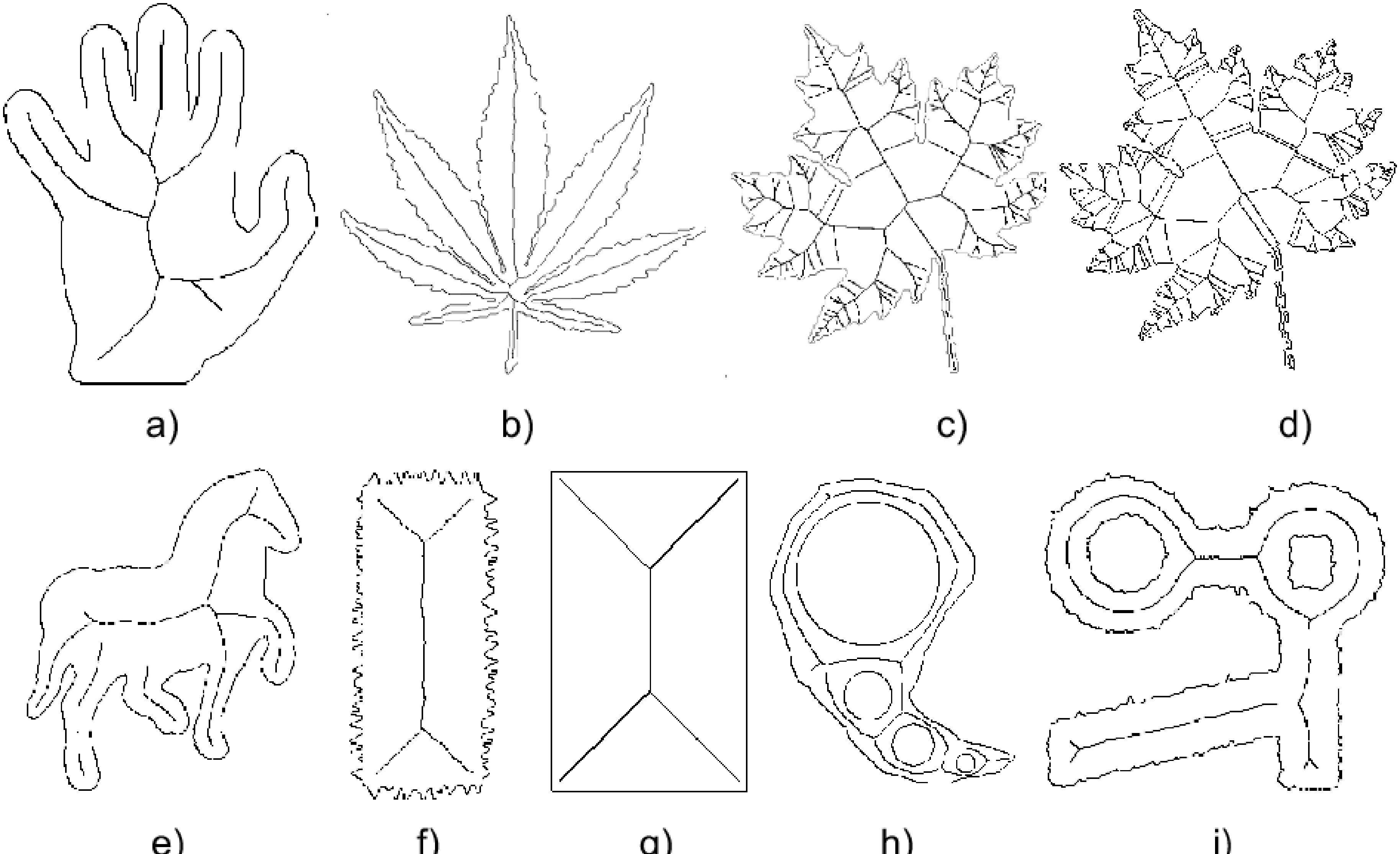
- Geometric analysis: aspect ratio, eccentricity, curvature and elongation
- Topological analysis: genus
- Retrieval: find the shape matching a source shape
- Classification: partition the shape into classes
- Matching: find the similarity between two shapes

Skeletonization

- Skeletons are the medial axes
- Skeletons are the set of points situated at equal distance from at least two boundary feature points of the given shape

$$S(\Omega) = \{ p \in \Omega \mid \exists q, r \in \partial\Omega, \left| p - q \right| = \left| p - r \right| \}$$

Skeletonization

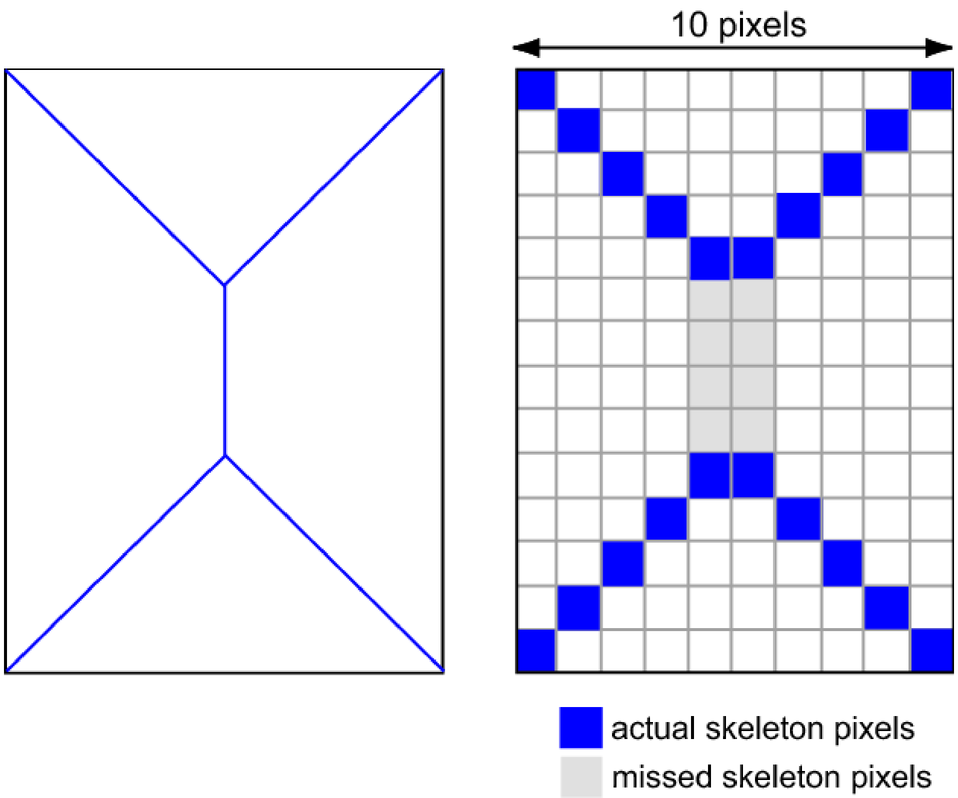


Skeleton Computation

Feature Transform Method:

Select those points whose feature transform contains more than two boundary points.

Works well on continuous data

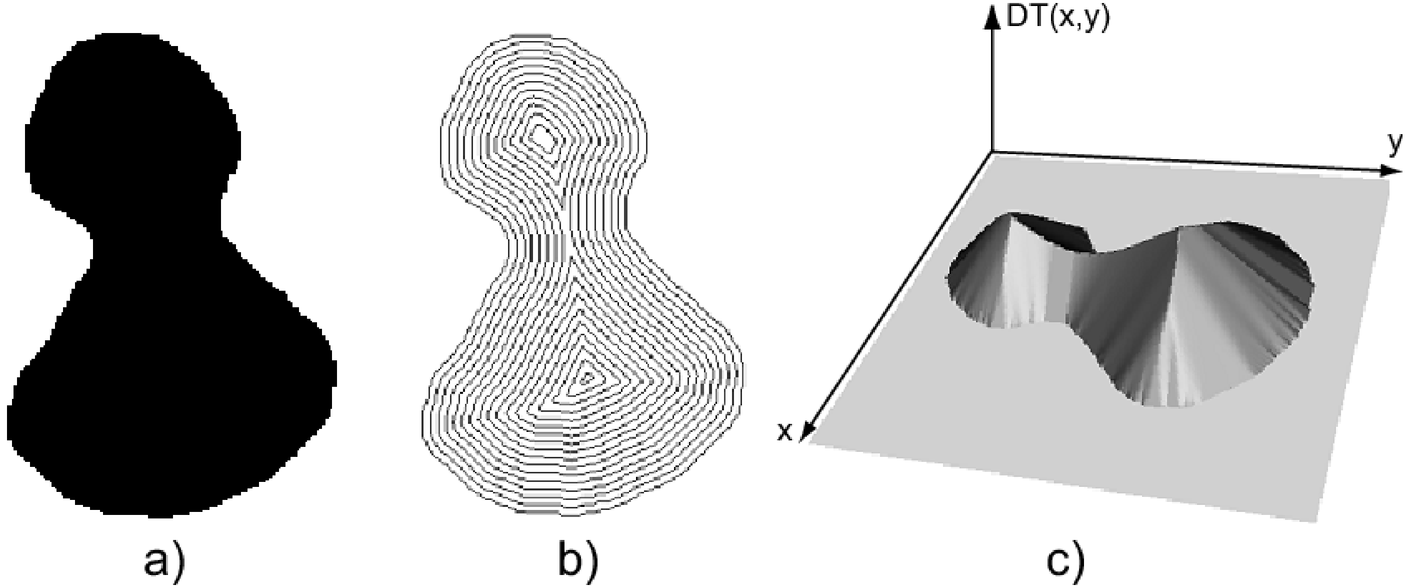


a) b)

Fails on discreate data

Skeleton Computation

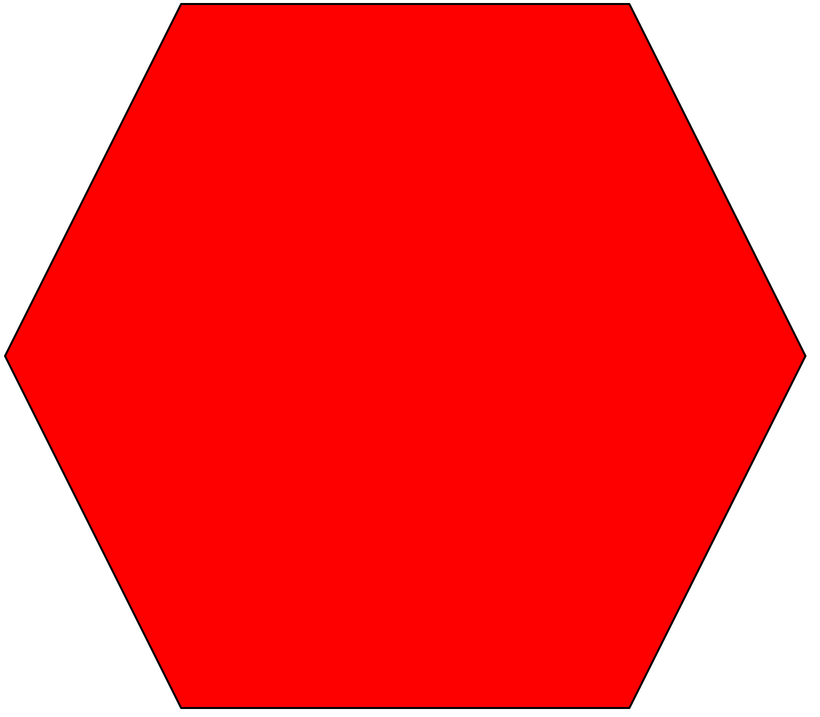
Using distance field singularities:
Skeleton points are local maxima of distance transform



MATLAB Tutorial

Video Demo

http://www.mathworks.com/videos/medical-imaging-analysis-and-visualization-71066.html



End
of Chap. 9

Note: covered all sections except 9.4.7 (skeleton in 3D)