

Jan. 31, 2013

①

* Example: line cells

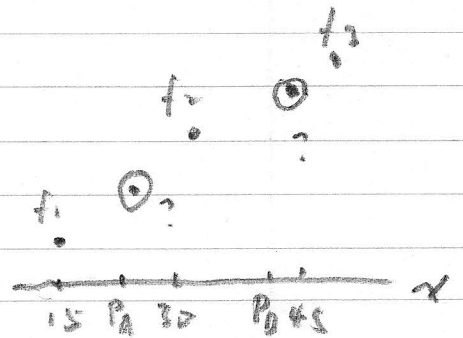
$$P_1: x_1 = 15, f_1 = 0.7$$

$$x_2 = 30, f_2 = 3.4$$

$$x_3 = 45, f_3 = 5.0$$

general formula

$$f(x) = \sum_{i=1}^2 f_i \phi_i(x)$$



(1) constant basis function

$$\phi_i(x) = \frac{1}{N} = \frac{1}{2}, \quad N: \text{number of vertices of the cell}$$

$$\begin{aligned} f_A &= f_1 \phi_1 + f_2 \phi_2 = \frac{1}{2}(f_1 + f_2) \\ &= \frac{1}{2}(0.7 + 3.4) = 2.05. \end{aligned}$$

The value is independent of the position in the cell

$$f_B = f_2 \phi_2 + f_3 \phi_3 = \frac{1}{2}(f_2 + f_3) = \frac{1}{2}(3.4 + 5.0) = 4.2$$

(2) linear basis function

$$f(x) = \sum_{i=1}^2 f_i \phi_i(x) = \sum_{i=1}^2 f_i \Psi_i(r)$$

$$r = T^{-1}(x)$$

$$r = \frac{(x - P_1) \cdot (P_2 - P_1)}{\|P_2 - P_1\|^2}$$

$$\Psi_1(r) = 1 - r$$

$$\Psi_2(r) = r$$

②

$$\text{For } P_A. \quad r = \frac{(x-x_1) \cdot (x_2-x_1)}{(x_2-x_1)^2} = \frac{x-x_1}{x_2-x_1}$$

$$r = \frac{22-15}{30-15} = \frac{7}{15}$$

$$f_A = f_1 \Phi_1(r) + f_2 \Phi_2(r)$$

$$f_A = 0.7 \cdot \left(1 - \frac{7}{15}\right) + 3.4 \cdot \frac{7}{15}$$

$$f_A = 1.96$$

$$\text{For } P_B. \quad r = \frac{(x-x_2) \cdot (x_3-x_2)}{(x_3-x_2)^2} = \frac{x-x_2}{x_3-x_2}$$

$$r = \frac{40-30}{45-30} = \frac{10}{15} = \frac{2}{3}$$

$$f_B = f_2 (1-r) + f_3 \cdot r$$

$$= 3.4 \cdot \frac{1}{3} + 5.2 \cdot \frac{2}{3}$$

$$f_B = 4.47$$

— Same thing for 2-D points

