Jan. 31, 2013

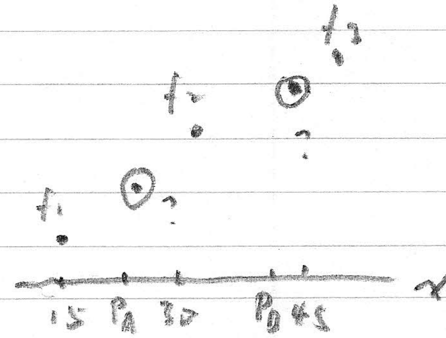
①

* Example: line cells

P_i : $x_1 = 15$, $f_1 = 0.7$

$x_2 = 30$, $f_2 = 3.4$

$x_3 = 45$, $f_3 = 5.0$



general formula

$$f_{(x)} = \sum_{i=1}^{2} f_i \phi_{i(x)}$$

(1) constant basis function

$$\phi_{i(x)} = \frac{1}{N} = \frac{1}{2} ,$$ N: number of vertices of the cell

$$f_A = f_1 \phi_1 + f_2 \phi_2 = \frac{1}{2}(f_1 + f_2)$$
$$= \frac{1}{2}(0.7 + 3.4) = 2.05.$$

The value is independent of the position in the cell

$$f_B = f_2 \phi_2 + f_3 \phi_3 = \frac{1}{2}(f_2 + f_3) = \frac{1}{2}(3.4 + 5.0) = 4.2$$

(2) Linear basis function

$$f(x) = \sum_{i=1}^{2} f_i \phi_{i(x)} = \sum_{i=1}^{2} f_i \Phi_i(r)$$

$$r = T^{-1}(x)$$

$$r = \frac{(\vec{P} - \vec{P_1}) \cdot (\vec{P_2} - \vec{P_1})}{\| \vec{P_2} - \vec{P_1} \|^2}$$

$$\Phi_1(r) = 1 - r$$
$$\Phi_2(r) = r$$

For P_A. $r = \frac{(x-x_1)\cdot(x_2-x_1)}{(x_2-x_1)^2} = \frac{x-x_1}{x_2-x_1}$

$r = \frac{22-15}{30-15} = \frac{7}{15}$

$f_A = f_1 \Phi_1(r) + f_2 \Phi_2(r)$

$f_A = 0.7 \cdot (1 - \frac{7}{15}) + 3.4 \cdot \frac{7}{15}$

$f_A = 1.96$

For P_B. $r = \frac{(x-x_2)(x_3-x_2)}{(x_3-x_2)^2} = \frac{x-x_2}{x_3-x_2}$

$r = \frac{40-30}{45-30} = \frac{10}{15} = \frac{2}{3}$

$f_B = f_2 (1-r) + f_3 \cdot r$

$= 3.4 \cdot \frac{1}{3} + 5.0 \cdot \frac{2}{3}$

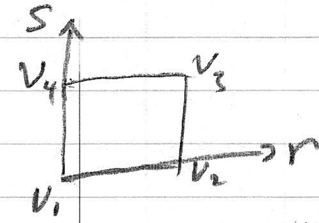
$f_B = 4.47$

— Same thing for 2-D points

* Example Quad cell

$$\tilde{f} = \sum_{i=1}^{4} f_i \phi_i(x, y, z) = \sum_{i=1}^{4} f_i \Phi_i(r, s, t)$$

$$\tilde{f} = \sum_{i=1}^{4} f_i \Phi_i(T^{-1}(x, y, z))$$

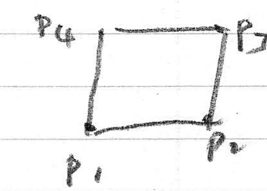


reference cell

(1) constant basis function

$$\Phi_i^0 = \frac{1}{N} = \frac{1}{4}$$

(2) linear basis function



Actual cell

$$\Phi_1^1(r, s) = (1-r)(1-s)$$

$$\Phi_2^1(r, s) = r(1-s)$$

$$\Phi_3^1(r, s) = rs$$

$$\Phi_4^1(r, s) = (1-r)s$$

$$r = \frac{(\vec{P} - \vec{P_1}) \cdot (\vec{P_2} - \vec{P_1})}{\| \vec{P_2} - \vec{P_1} \|^2}$$

$$s = \frac{(\vec{P} - \vec{P_1}) . (\vec{P_4} \cdot \vec{P_1})}{\| \vec{P_4} - \vec{P_1} \|^2}$$

(1) constant basis function

$$f_A = (f_1 + f_2 + f_3 + f_4)/4$$

$$= (1.0 + 1.2 + 1.5 + 1.5)/4 = 1.3$$

(2) Linear basis function

$$\vec{p} - \vec{p}_1 = \begin{bmatrix} 1.5 \\ 1.2 \\ 1.3 \end{bmatrix} - \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.3 \end{bmatrix}$$

$$\vec{p}_2 - \vec{p}_1 = \begin{bmatrix} 2.0 \\ 1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0 \\ 0 \end{bmatrix}$$

$$\|\vec{p}_2 - \vec{p}_1\|^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 = 1^2 + 0^2 + 0^2 = 1$$

$$(\vec{p} - \vec{p}_1) \cdot (\vec{p}_2 - \vec{p}_1) = 0.5 \times 1.0 + 0.2 \times 0 + 0.3 \times 0 = 0.5$$

$$r = \frac{(\vec{p} - \vec{p}_1) \cdot (\vec{p}_2 - \vec{p}_1)}{\|\vec{p}_2 - \vec{p}_1\|^2} = \frac{0.5}{1} = 0.5$$

Similarly, for s

$$\vec{p}_4 - \vec{p}_1 = \begin{bmatrix} 1.0 \\ 2.0 \\ 1.8 \end{bmatrix} - \begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0.8 \end{bmatrix}$$

$$\|\vec{p}_4 - \vec{p}_1\|^2 = 0^2 + 1^2 + 0.8^2 = 1.64$$

$$s = \frac{(\vec{p} - \vec{p}_1) \cdot (\vec{p}_4 - \vec{p}_1)}{\|\vec{p}_4 - \vec{p}_1\|^2} = \frac{0.44}{1.64} = 0.27$$

$$(\vec{p} - \vec{p}_1) \cdot (\vec{p}_4 - \vec{p}_1) = 0.5 \times 0 + 0.2 \times 1 + 0.3 \times 0.8 = 0.44$$

Calculate Φ_i:

$$\Phi_1 = (1-r)(1-s) = (1-0.5)(1-0.27) = 0.37$$

$$\Phi_2 = r(1-s) = 0.5(1-0.27) = 0.37$$

$$\Phi_3 = rs = 0.5 \times 0.27 = 0.14$$

$$\Phi_4 = (1-r) \cdot s = (1-0.5) \cdot 0.27 = 0.14$$

Finally, $f_A = f_1 \Phi_1 + f_2 \Phi_2 + f_3 \Phi_3 + f_4 \Phi_4$

$$f_A = 1.0 \times 0.37 + 1.2 \cdot 0.37 + 1.5 \cdot 0.14 + 1.5 \times 0.14 = \boxed{1.23}$$