CDS 301 Fall 2013
Scientific Information and Data Visualization

Homework Assignment 8

Assignment Date: April 17, 2013
Due Date: April 23, 2013

1. Surface Reconstruction from scattered points

One important operation to reconstruct a 3D surface from a point cloud is to compute a tangent plane that approximate the surface in the neighborhood of a point. The plane is defined by its center and normal. Considering the data point P0=(0,0,0), it has 8 neighboring points within the support radius; these points are

P1=(1.0, 0.0, 0.5)
P2=(1.0, 0.8, 0.4)
P3=(0.0, 1.2, 0.6)
P4=(-0.5, 1.0, 0.3)
P5=(-1.0, 0.1, 0.1)
P6=(-0.7, -0.5, -0.3)
P7=(0.0, -1.0, -0.5)
P8=(0.9, -0.6, -0.4)

(1) calculate the center C=(Cx, Cy, Cz) of these points [0.0875, 0.125, 0.0875]
(2) calculate the 3 X 3 covarience matrix of these points
(3) find the normal direction of the tangent plane of these points. The normal is the eigenvector corresponding to the smallest eigenvalue of the covariance matrix.]

Note: To obtain the eigenvalues and eigenvectors of the 3 x 3 matrix, use Matlab "eig" function, e.g., >>[v, d] = eig(H)

Answer:

(1)

$$Cx = \frac{\sum_{i=1}^{8} p_i^x}{8} = \frac{1.0 + 1.0 + 0.0 - 0.5 - 1.0 - 0.7 + 0 + 0.9)}{8} = 0.0875$$

Similarly,

$Cy = 0.125$

$Cz = 0.0875$

(2)

$$A = (a_{jk}) = \begin{pmatrix} a_{11}, a_{12}, a_{13} \\ a_{21}, a_{22}, a_{23} \\ a_{31}, a_{32}, a_{33} \end{pmatrix}$$

$$a_{11} = \sum_{i=1}^{8} (p_i^x - Cx)(p_i^x - Cx) = 4.489$$

$$a_{12} = \sum_{i=1}^{8} (p_i^x - Cx)(p_i^y - Cy) = -0.078$$

$$a_{13} = \sum_{i=1}^{8} (p_i^x - Cx)(p_i^z - Cz) = 0.439$$

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$$A = \begin{pmatrix} 4.489, -0.078, 0.439 \\ -0.078, 4.575, 2.153 \\ 0.439, 2.153, 1.309 \end{pmatrix}$$

3. Run Matlab, >>[v, d] = eig(H)
The three eighevalues: 0.197, 4.517, 5.658
The eigenvector of the smallest eigenvector is (-0.099, - 0.440, 0.892); this is the normal direction

2. Edge Detection

Edge detection requires the calculation of image derivative. Considering the point P(5,5) in a 100 X 100 pixel images, the values of the point and its eight neighboring points are:

P(4,4)=1.2; P(4,5)=1.7; P(4,6)=2.0
P(5,4)=1.5; P(5,5)=2.0; P(5,6)=2.2
P(6,4)=1.9; P(6,5)=2.5; P(6,6)=3.0

(1) calculate the derivative at P(5,5) using the standard derivative method (formula 9.10)? 0.539
(2) calculate the derivative at P(5,5) using Roberts operator (formula 9.11)? 1.04
(3) calculate the derivative at P(5,5) using Sobel operator (formula 9.12)? 4.67
(4) calculate the Laplacian at P(5,5) using Laplacian operator (formula 9.14)? 0.10

Answer:

Answer
(1) Standard

$$\frac{\partial I}{\partial x}(5,5) = I(6,5) - I(5,5) = 2.5 - 2.0 = 0.5$$

$$\frac{\partial I}{\partial y}(5,5) = I(5,6) - I(5,5) = 2.2 - 2.0 = 0.2$$

$$\left|\nabla I(5,5)\right| = \sqrt{0.5^2 + 0.2^2} = 0.539$$

(2) Roberts

$$R(5,5) = \sqrt{(I_{6,6} - I_{5,5})^2 + (I_{6,5} - I_{5,6})^2} = \sqrt{(3.0 - 2.0)^2 + (2.5 - 2.2)^2} = 1.04$$

(3) Sobel

$$\frac{\partial I}{\partial x}(5,5) = I(6,4) + 2I(6,5) + I(6,6) - I(4,4) - 2I(4,5) - I(4,6)$$

$$= 1.9 + 2 * 2.5 + 3.0 - 1.2 - 2 * 1.7 - 2.0 = 3.3$$

$$\frac{\partial I}{\partial y}(5,5) = I(6,6) + 2I(5,6) + I(4,6) - I(6,4) - 2I(5,4) - I(4,4)$$

$$= 3.0 + 2 * 2.2 + 2.0 - 1.9 - 2 * 1.5 - 1.2 = 3.3$$

$$\left|\nabla I(5,5)\right| = \sqrt{(3.3)^2 + (3.3)^2} = 4.67$$

(4) Laplacian

$$\Delta I(5,5) = 4I(5,5) - I(6,5) - I(4,5) - I(5,6) - I(5,4)$$

$$= 4 * 2.0 - 2.5 - 1.7 - 2.2 - 1.5 = 0.10$$