# CDS 301 Spring 2013
# Scientific Information and Data Visualization

## Homework #7 - Solution

**Assignment Date: April 02, 2013**
**Due Date: April 09, 2013**

**1. Curvature tensor**

For the continuous function z=sin(x)cos(y)

(1) Obtain its gradient at points P1=(0,0) and P2=(1,1)? **(1, 0) and (0.29, -0.71)**

(2) Obtain its Hessian matrix at points P1=(0,0) and P2=(1,1)?

$$H = \begin{pmatrix} 0,0 \\ 0,0 \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} -0.45,-0.45 \\ -0.45,-0.45 \end{pmatrix}$$

(3) For point P1=(0,0), calculate the curvatures along the direction S with α=0° and α=60° respectively, where α is the angle between the direction vector S and the X-axis. **0 and 0**

(4) Repeat the tasks in (3) for the point at P2=(1,1)? **-0.45 and -0.85**

**Answer:**

(1) obtain its gradient at points P1=(0,0) and P2=(1,1)

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (\cos(x)\cos(y), -\sin(x)\sin(y))$$

$$P1: \nabla f = (1\times 1, -0\times 0) = (1,0)$$

$$P2: \nabla f = (0.54\times 0.54, -0.84\times 0.84) = (0.29, -0.71)$$

(2) obtain its Hessian matrix at points P1=(0,0) and P2=(1,1), respectively

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2} \end{pmatrix} = \begin{pmatrix} -\sin(x)\cos(y), -\cos(x)\sin(y) \\ -\cos(x)\sin(y), -\sin(x)\cos(y) \end{pmatrix}$$

$$P1 = (0,0)$$

$$H = \begin{pmatrix} -0\times 1, -1\times 0 \\ -1\times 0, -0\times 1 \end{pmatrix} = \begin{pmatrix} 0,0 \\ 0,0 \end{pmatrix}$$

$$P1 = (1,1)$$

$$H = \begin{pmatrix} -0.84\times 0.54, -0.54\times 0.84 \\ -0.54\times 0.84, -0.84\times 0.54 \end{pmatrix} = \begin{pmatrix} -0.45, -0.45 \\ -0.45, -0.45 \end{pmatrix}$$

(3) For point P1=(0,0), calculate the curvatures along the direction S with α=0° and α=60° respectively, where α is the angle between the direction vector S and the X-axis.

$$s = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$cur = s^T H s = (\cos\alpha, \sin\alpha) \begin{pmatrix} \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2} \end{pmatrix} \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$\alpha = 0°$$

$$s = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$cur = (1,0) \begin{pmatrix} 0,0 \\ 0,0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (0,0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$$

$$\alpha = 60° = 1.05$$

$$s = \begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix}$$

$$cur = (0.50, 0.87) \begin{pmatrix} 0,0 \\ 0,0 \end{pmatrix} \begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix} = (0,0) \begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix} = 0$$

(4) Repeat the task in (3) for the point P2=(1,1)

$$\alpha = 0°$$

$$s = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$cur = (1,0)\begin{pmatrix} -0.45,-0.45 \\ -0.45,-0.45 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = (-0.45,-0.45)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -0.45$$

$$\alpha = 60° = 1.05$$

$$s = \begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix}$$

$$cur = (0.50,0.87)\begin{pmatrix} -0.45,-0.45 \\ -0.45,-0.45 \end{pmatrix}\begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix}$$

$$= (-0.62,-0.62)\begin{pmatrix} 0.50 \\ 0.87 \end{pmatrix} = -0.85$$

**2. Eigenvalue and Eigenvector.**

For the given matrix that characterize a tensor in 2-D,

$$H = \begin{pmatrix} 1, & 3 \\ 2, & 2 \end{pmatrix}$$

(1) Calculate its eigenvalues? **4 and -1**

(2) Calculate the corresponding eigenvectors? **(0.707, 0.701) and (-0.832, 0.555)**

or $$\begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} \text{ and } \begin{pmatrix} -\frac{3\sqrt{13}}{13} \\ \frac{2\sqrt{13}}{13} \end{pmatrix}$$

**Answer:**

$$\det(H-\lambda I)=0$$

$$\det\begin{pmatrix}1-\lambda,3\\2,2-\lambda\end{pmatrix}=0$$

$$(1-\lambda)(2-\lambda)-6=0$$

$$\lambda^2-3\lambda-4=0$$

$$\lambda_1=4$$

$$\lambda_2=-1$$

$$For\ \lambda_1$$

$$Hs=\lambda_1 s$$

$$\begin{pmatrix}1, & 3\\2, & 2\end{pmatrix}\begin{pmatrix}s_x\\s_y\end{pmatrix}=4\begin{pmatrix}s_x\\s_y\end{pmatrix}$$

$$s_x=s_y$$

$$also\quad {s_x}^2+{s_y}^2=1$$

$$s_x=\frac{\sqrt{2}}{2}$$

$$s_y=\frac{\sqrt{2}}{2}$$

$$s_1=\begin{pmatrix}\frac{\sqrt{2}}{2}\\\frac{\sqrt{2}}{2}\end{pmatrix}$$

$For \quad \lambda_2$

$Hs = \lambda_2 s$

$$\begin{pmatrix} 1, & 3 \\ 2, & 2 \end{pmatrix} \begin{pmatrix} s_x \\ s_y \end{pmatrix} = -1 \begin{pmatrix} s_x \\ s_y \end{pmatrix}$$

$$s_x = -\frac{3}{2} s_y$$

$also \quad s_x^{\ 2} + s_y^{\ 2} = 1$

$$s_y = \frac{2\sqrt{13}}{13}$$

$$s_x = -\frac{3\sqrt{13}}{13}$$

$$s_1 = \begin{pmatrix} -\frac{3\sqrt{13}}{13} \\ \frac{2\sqrt{13}}{13} \end{pmatrix}$$