CDS 301 Spring 2013
Scientific Information and Data Visualization

Homework Assignment 1 - Answer

Assignment Date: January 29, 2013
Due Date: February 05, 2013

1 **(50 pts). Basis Function and coordinate transform in 1-D**

Considering the following two cells, C_1 and C_2, of the line-segment type in 1-D function domain: $C_1=\{P_1,P_2\}$, $C_2=\{P_2, P_3\}$, where $P_1=1$, $P_2=2$, and $P_3=3$ respectively. The sampling values at the three sampling points are $f_1=0.2$, $f_2=0.4$ and $f_3=0.9$, respectively. You are asked to reconstruct the data values using interpolation at points $P_A=1.3$ and $P_B=2.8$

(1) Find f_A and f_B using a constant basis function **(0.3; 0.65)**

(2) Find f_A and f_B using a linear basis function **(0.26; 0.80)**

Answer: For a two-vertex cell, the reconstruction function can be written as

$$f(x) = \sum_{i=1}^{2} f_i \phi_i(x)$$

(1) Constant basis function. It is equivalent to find the value at the center of the cell, and then all the points within the cell are equal to this value. The weight from all vertices are equal, that is, the basis function is

$$\phi_i(x) = 1/N = 1/2$$

$$f(x) = \frac{f_1}{2} + \frac{f_2}{2}$$

For point A within cell C_1, $f_A = 0.2/2.0+0.4/2.0 = 0.3$
For point B within cell C_2, $f_B = 0.4/2.0+0.9/2.0 = 0.65$

(2) Linear basis function. The functional values at the two vertices are weighted differently, depending on the distance from the point to the vertices. The basis function is now a linear function of such a distance. Since the basis function is defined in the reference cell, we also need to convert the distance in the world or actual cell to that in the reference cell. The mathematic formulation can be written as:

$$f(x) = \sum_{i=1}^{2} f_i \phi_i(x) = \sum_{i=1}^{2} f_i \Phi_i(r) = f_1\Phi_1(r) + f_2\Phi_2(r)$$

$$r = T^{-1}(x) = \frac{(\vec{p} - \vec{p}_1)\cdot(\vec{p}_2 - \vec{p}_1)}{\| \vec{p}_2 - \vec{p}_1 \|^2}$$

$$\Phi_1(r) = 1 - r$$

$$\Phi_2(r) = r$$

For point A, $r = \dfrac{(1.3-1)\cdot(2-1)}{(2-1)^2} = 0.3$

$$\Phi_1 = 1 - 0.3 = 0.7$$
$$\Phi_2 = 0.3$$

$f_A = 0.2 \times 0.7 + 0.4 \times 0.3 = 0.26$

For point B, $r = \dfrac{(2.8-2)\cdot(3-2)}{(3-2)^2} = 0.8$

$$\Phi_1 = 1 - 0.8 = 0.2$$
$$\Phi_2 = 0.8$$

$f_B = 0.4 \times 0.2 + 0.9 \times 0.8 = 0.80$

2 (50 pts). Basis Function and coordinate transform in 3-D

Considering an actual cell of quad type in 3-D space, C1={$\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, $\mathbf{P_4}$}, where $\mathbf{P_1}$=[0.0, 0.0, 0.0] and $\mathbf{P_2}$=[1.0, 0.0, 2.0], $\mathbf{P_3}$=[1.0, 3.0 ,2.0], and $\mathbf{P_4}$=[0.0, 3.0, 0.0], The functional values of the four vertex points are f_1=5.0, f_2=6.0, f_3=7.0 and f_4=8.0. You are asked to reconstruct the functional value at point $\mathbf{P_A}$ and $\mathbf{P_B}$, where $\mathbf{P_A}$=[0, 0, 0], and $\mathbf{P_B}$=[0.5, 2.0, 1.0]

(1) To find f_A and f_B, use constant basis function. **(6.5; 6.5)**

(2) To find f_A and f_B, use linear basic function. **(5.0, 6.83)**

Note: You need to write down the formulas first, before plugging in the numbers and calculation.

Answer: For a four-vertex quad cell, the reconstruction function can be written as

$$f(x) = \sum_{i=1}^{4} f_i \phi_i(x)$$

(1) Constant basis function. It is equivalent to find the value at the center of the cell, and then all the points within the cell are equal to this value. The weight from all vertices are equal, that is, the basis function is

$$\phi_i(x) = 1/N = 1/4$$

$$f(x) = \frac{f_1}{4} + \frac{f_2}{4} + \frac{f_3}{4} + \frac{f_4}{4}$$

For point A within the cell, $f_A = (5.0 + 6.0 + 7.0 + 8.0) / 4 = 6.50$

For point B within the cell, $f_B = f_A = (5.0 + 6.0 + 7.0 + 8.0) / 4 = 6.50$

(2) Linear basis function. The functional values at the four vertices are weighted differently, depending on the distances from the point to the vertices. The basis function is now a linear function of such distances. Since the basis function is defined in the reference cell, we also need to convert the distances in the world or actual cell to that in the reference cell. Since it is a

topologically 2-D reference cell, there are two distances (r,s) that need to be defined. The mathematic formulation can be written as:

$$f(x,y,z)=\sum_{i=1}^{4} f_i\phi_i(x,y,z)=\sum_{i=1}^{4} f_i\Phi_i(r,s)$$
$$f(x,y,z)=f_1\Phi_1(r,s)+f_2\Phi_2(r,s)+f_3\Phi_3(r,s)+f_4\Phi_4(r,s)$$
$$r=T^{-1}(x,y,z)=\frac{(\vec{p}-\vec{p}_1)\cdot(\vec{p}_2-\vec{p}_1)}{\|\vec{p}_2-\vec{p}_1\|^2}$$
$$s=T^{-1}(x,y,z)=\frac{(\vec{p}-\vec{p}_1)\cdot(\vec{p}_4-\vec{p}_1)}{\|\vec{p}_4-\vec{p}_1\|^2}$$
$$\Phi_1(r,s)=(1-r)(1-s)$$
$$\Phi_2(r,s)=r(1-s)$$
$$\Phi_3(r,s)=rs$$
$$\Phi_4(r,s)=(1-r)s$$

For point A,

To find r :

$$\vec{p}-\vec{p}_1=(0.0,0.0,0.0)-(0.0,0.0,0.0)=(0.0,0.0,0.0)$$
$$\vec{p}_2-\vec{p}_1=(1.0,0.0,2.0)-(0.0,0.0,0.0)=(1.0,0.0,2.0)$$
$$(\vec{p}-\vec{p}_1)\cdot(\vec{p}_2-\vec{p}_1)=0.0\times1.0+0.0\times0.0+0.0\times2.0=0$$
$$\|\vec{p}_2-\vec{p}_1\|^2=1.0^2+0.0^2+2.0^2=5.0$$
$$r=\frac{0}{5}=0.0$$

To find s :

$$\vec{p}-\vec{p}_1=(0.0,0.0,0.0)-(0.0,0.0,0.0)=(0.0,0.0,0.0)$$
$$\vec{p}_4-\vec{p}_1=(0.0,3.0,0.0)-(0.0,0.0,0.0)=(0.0,3.0,0.0)$$
$$(\vec{p}-\vec{p}_1)\cdot(\vec{p}_4-\vec{p}_1)=0.0\times0.0+0.0\times3.0+0.0\times0.0=0$$
$$\|\vec{p}_4-\vec{p}_1\|^2=0.0^2+3.0^2+0.0^2=9.0$$
$$s=\frac{0}{9}=0.0$$

Thus, $\Phi_1=1$

$$\Phi_2=\Phi_3=\Phi_4=0$$

$$f_A=5.0\times1.0+6.0\times0.0+7.0\times0.0+8.0\times0.0=5.0$$

The functional value at point A is the same as that at P_1. This all makes sense in a linear interpolation. Since point A coincides with P_1, the functional values should be the same.

For point B,

To find r :

$$\vec{p} - \vec{p}_1 = (0.5,2.0,1.0) - (0.0,0.0,0.0) = (0.5,2.0,1.0)$$

$$\vec{p}_2 - \vec{p}_1 = (1.0,0.0,2.0) - (0.0,0.0,0.0) = (1.0,0.0,2.0)$$

$$(\vec{p} - \vec{p}_1) \cdot (\vec{p}_2 - \vec{p}_1) = 0.5 \times 1.0 + 2.0 \times 0.0 + 1.0 \times 2.0 = 2.5$$

$$\| \vec{p}_2 - \vec{p}_1 \|^2 = 1.0^2 + 0.0^2 + 2.0^2 = 5.0$$

$$r = \frac{2.5}{5} = 0.5$$

To find s :

$$\vec{p} - \vec{p}_1 = (0.5,2.0,1.0) - (0.0,0.0,0.0) = (0.5,2.0,1.0)$$

$$\vec{p}_4 - \vec{p}_1 = (0.0,3.0,0.0) - (0.0,0.0,0.0) = (0.0,3.0,0.0)$$

$$(\vec{p} - \vec{p}_1) \cdot (\vec{p}_4 - \vec{p}_1) = 0.5 \times 0.0 + 2.0 \times 3.0 + 1.0 \times 0.0 = 6.0$$

$$\| \vec{p}_4 - \vec{p}_1 \|^2 = 0.0^2 + 3.0^2 + 0.0^2 = 9.0$$

$$s = \frac{6.0}{9.0} = \frac{2}{3}$$

Thus,

$$\Phi_1 = (1 - 0.5)(1 - 2/3) = 1/6$$

$$\Phi_2 = 0.5(1 - 2/3) = 1/6$$

$$\Phi_3 = 0.5(2/3) = 1/3$$

$$\Phi_3 = (1 - 0.5)(2/3) = 1/3$$

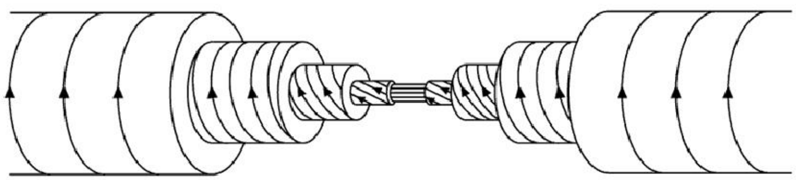
$$f_A = 5.0 \times 1/6 + 6.0 \times 1/6 + 7.0 \times 1/3 + 8.0 \times 1/3 = 6.83$$

3. Your favorite visualization application

In a separate piece of paper, write down your idea of a particular visualization application. This application could be your favorite one, given your interest in a particular scientific discipline. The application should have a clear visualization objective, as well as the scientific justification and detailed specification on the requirements. The following is one example of my own favorite application.

Example:

My visualization goal is to draw an object named magnetic flux rope. A magnetic flux rope has a 3-D cylindrical shape, with helical magnetic field lines wrapping around the surface of the cylinder. You are required to draw the cylindrical surface as well as the helical magnetic field lines, in order to fully illustrate the structure of a flux rope. One simple version of the visualization of the flux rope is illustrated in Figure 1.



Interior Structure of Flux Rope

Figure: Structure of a flux rope

A Magnetic flux rope is an important scientific phenomena or object that occurs in many places, including in the Sun's atmosphere, the space between planets, and in the laboratories. It is composed of organized magnetic fields and electric currents, and thus carries a large amount of energy with it. It is the primary driver of severe space weather that affect society and life.

The mathematic equation controlling the flux rope is the Bessel differential equation as follows:

$$\nabla^2 \vec{B} + \alpha^2 \vec{B} = 0$$

Where **B** is the vector describing the magnetic field, α is a constant quantifying the magnitude of the helical shape (or equivalently, the strength of electric current, or energy). The mathematic solution to this equation in a cylindrical geometry is a set of three scalar functions that described the r (radial), φ (azimuth), z (axial) components of the magnetic field, respectively:

$$B_r = 0$$

$$B_\varphi = B_0 J_1(\alpha r)$$

$$B_z = B_0 J_0(\alpha r)$$

Where both B_0 and α are constants, and can be chosen as value 1. J_0 and J_1 are the zeroth and first order Bessel functions, respectively.