Dec. 4, 2012

stellar Atmosphere (CH4)

* A simple Atmosphere (CH 4.3)
  — gray atmosphere ; $I_\nu = I$
  Approximation of being independent of ν
* Line profiles and curve of Growth (CH 4.8)
  — how to determine the strength of a line. $\boxed{I_\nu}$

A simple Atmosphere

* what is photospheric surface R?
  what is τ_p at R?
* what is true surface? $\tau = 0$
* Prove: $T^4(\tau) = \frac{1}{2} T_{eff}^4 (1 + \frac{3}{2}\tau)$ in the atmosphere

* Importance of studying atmosphere
  1. provide the correct boundary for stellar models
  2. where observations can be made

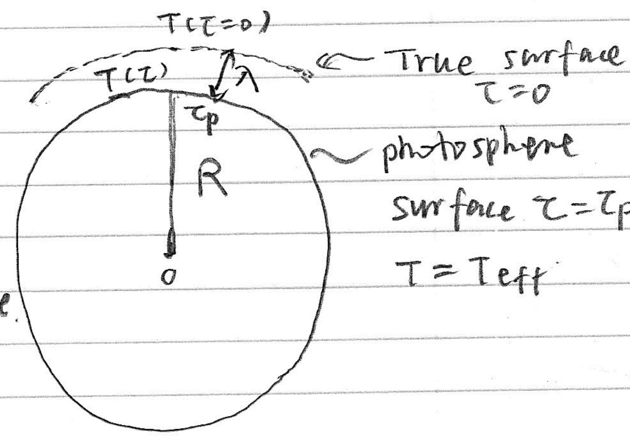
Find $T(\tau)$

* photospheric surface
  where all radiation comes from

$$L = 4\pi R^2 \sigma T_{eff}^4$$

Both L and T_{eff} are observable.

R can be inferred.



At R: $T = T_{eff}$, $\tau = \tau_p$, $\rho = \rho_p$; $I(\theta)$: near isotropic

$\tau < \tau_p$: below the photosphere, no radiation emerges

Thus, $\tau_p \approx 1$ (show later, $\tau_p = \frac{2}{3}$)

$\tau > \tau_p$: give the observed radition.

* True surface.

At $R+\lambda$: λ; thickness of the photosphere
($\lambda \sim 100$ km) for the Sun)

* No incoming radiation $I(\frac{\pi}{2} \le \theta \le \pi) = 0$

or $I(\mu) = 0$ for $-1 \le \mu \le 0$

* $P_g \sim 0$: gas pressure close to 0 because $\rho_g \to 0$
* $P = P_{rad}(\tau=0) \neq 0$: "zero-boundary" is inaccurate
* In the photosphere λ $(R \le r \le R+\lambda)$
  radiation pressure dominates the gas pressure

* Let's consider P_r.

In LTE: $P_r = \frac{aT^4}{3}$ dyne cm^{-2}

$$I = B = \frac{\sigma T^4}{\pi} = \frac{ac}{4\pi} T^4 \quad ; \quad \sigma = \frac{ac}{4}$$

$$F = 2\pi \int_{-1}^{+1} I(\mu)\, \mu\, d\mu$$

$$u_r = 2\pi \int_{-1}^{+1} \frac{I(\mu)}{c}\, d\mu = \frac{4\pi}{c} I = \frac{4\pi}{c} \cdot \frac{\sigma T^4}{\pi} = aT^4.$$

$$P_r = 2\pi \int_{-1}^{+1} \frac{I(\mu)}{c} \mu^2 d\mu = \frac{2\pi I}{c} \underbrace{\int_{-1}^{+1} \mu^2 d\mu}_{} \quad \frac{2}{3}$$

$$P_r = \frac{4\pi}{3c} B = \frac{\cancel{4\pi}}{3c} \cdot \frac{acT^4}{\cancel{4\pi}} = \frac{a}{3} T^4$$

Note: for $\frac{I(\mu)}{c} \mu^2$: the integrant

$I(\mu)$: energy flux

$\frac{I(\mu)}{c}$: momentum flux.

since $E = pc$ for relativistic particles

μ: project the momentum to the normal direction

μ: project the transfer speed to normal direction

In the star, $F = \frac{4\pi}{3} \frac{\partial B}{\partial \tau}$ due to slight asymmetry $I(\theta)$,

For pure LTE, $F = 0$, $F_{in} = F_{out}$

For star near LTE, $F_{out} = B + \frac{2\pi}{3} \frac{\partial B}{\partial \tau}$

$$F_{in} = B - \frac{2\pi}{3} \frac{\partial B}{\partial \tau}$$

$$F = F_{in} + F_{out} = \frac{4\pi}{3} \frac{\partial B}{\partial \tau}$$

$$\partial \tau = -\kappa \rho \, dr \quad \text{(the defintion of } \tau\text{)}$$

$$F_\nu = -\frac{4\pi}{3} \frac{1}{\kappa_\nu \rho} \frac{\partial B_\nu}{\partial r} = \frac{L_\nu}{4\pi r^2} \quad \text{--- } 4.31$$

$$P_{r,\nu} = \frac{4\pi}{3c} B_\nu \quad \text{--- } 4.32$$

$$\frac{dP_{r,\nu}}{dr} = \frac{4\pi}{3c} \frac{dB_\nu}{dr}$$

From (4.31) $$\frac{dB_\nu}{dr} = -\frac{3}{16\pi^2} \kappa_\nu \rho \frac{L_\nu}{r^2}$$

$$\Rightarrow \frac{dP_{r,\nu}}{dr} = -\frac{\kappa_\nu \rho}{4\pi r^2 c} L_\nu \quad \text{--- } 4.34$$

Integrate over frequency

$$\int_0^\infty \frac{dP_{r,\nu}}{dr} d\nu = -\frac{\rho}{4\pi r^2 c} \int_0^\infty \kappa_\nu L_\nu \, d\nu$$

$$\Rightarrow \frac{dP_r}{dr} = -\frac{\kappa\rho}{4\pi r^2 c} L \qquad \text{--- } 4.34$$

where $\kappa = \dfrac{\int_0^\infty \kappa_\nu L_\nu \, d\nu}{L}$, $L = \int_0^\infty L_\nu d\nu$

κ: Differential luminosity-weighted opacity

For the atmosphere $r \sim R$

$$\frac{dP_r}{dr} = -\frac{\kappa\rho}{4\pi R^2 c} L$$

$$dP_r = \frac{L}{4\pi R^2 c} d\tau$$

Integrate from the true surface $\tau = 0$ to arbitrary τ

$$\int_0^\tau dP_r = \frac{L}{4\pi R^2 c} \tau$$

$$P_r(\tau) = \frac{L}{4\pi R^2 c} \tau + P_r(\tau=0)$$

$$P_r(\tau) = \frac{\sigma T_{eff}^4}{c} \tau + P_r(\tau=0) \qquad \text{--- } (4.38)$$

* what is $P_r(\tau=0)$?

Radiation pressure at the true surface is not zero, since outgoing radiation remains

$$P_r(\tau=0) = 2\pi \int_0^{+1} \frac{I(\tau=0)}{c} \mu^2 d\mu = \frac{2\pi}{3c} I(\tau=0) \quad -4.40$$

Since $L = 4\pi R^2 F_{out} = 4\pi R^2 \cdot 2\pi \int_0^{+1} I(\tau=0) \, \mu d\mu$

$$L = 4\pi R^2 \cdot \pi I(\tau=0) \qquad \text{--- } 4.41$$

$$\Rightarrow I(\tau=0) = \frac{L}{4\pi R^2} \frac{1}{\pi} = \frac{\sigma T_{eff}^4}{\pi}$$

Thus $P_r(\tau=0) \approx \frac{2}{3} \frac{\sigma T_{eff}^4}{c}$

$P_r(\tau) \approx \frac{\sigma T_{eff}^4}{c} (\tau + \frac{2}{3})$

At depth τ, apply LTE. $P_r(\tau) = \frac{1}{3} a T^4 = \frac{4\sigma}{3c} T^4$

$\Rightarrow T^4 = \frac{3}{4} (\tau + \frac{2}{3}) T_{eff}^4$

or $\boxed{T^4 = \frac{1}{2} T_{eff}^4 (1 + \frac{3}{2}\tau)}$ ---- (4.44)

* $\tau = \tau_p = \frac{2}{3}$, $T = T_{eff}$, the photosphere surface

* $\tau = 0$, $T = 2^{-\frac{1}{4}} T_{eff}$, the true surface

Thus, T is also not zero at true surface

$\boxed{T = 0.84\, T_{eff}}$ at $\tau = 0$

For the Sun, $T_{eff} = 5780$ K

$T(\tau=0) = 4855$ K.

or ~ 1000 K lower

* λ: thickness of photosphere

$\tau_p = \kappa \rho \lambda$

$\tau_p = \frac{2}{3}$; $\kappa = 1\ cm^2\ g^{-1}$; $\rho = 10^{-7}\ g\ cm^{-3}$

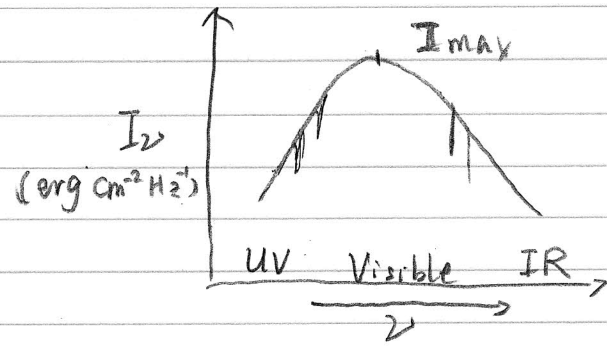
$\Rightarrow \lambda = 10^7$ cm $= 100$ km.

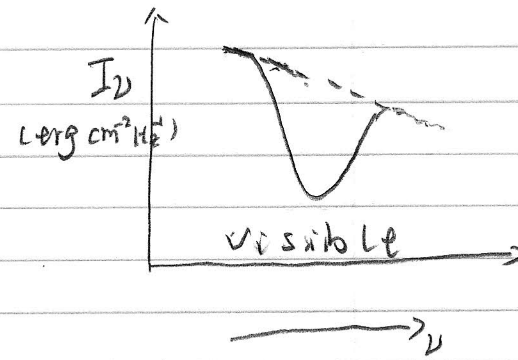
$\Rightarrow$ An extremely sharp atmosphere,
can't be resolved by a telescope

$\frac{\lambda}{R_\odot} = \frac{100\ km}{0.7 \times 10^6\ km} = 10^{-4} = 0.01\%$

stellar spectrum

— continuum + absorption spectrum





Continuum spectrum + lines (Low resolution observation)

Line profile (high resolution observ.)

$I_\nu = I_c + I_e$; $I_e < 0$ (absorption)

$I_{(e)} > 0$ (emission)

For photosphere, always absorption line.
because hotter and stronger radiation at depth

$$I_c = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

* planck function as derived from the distribution function

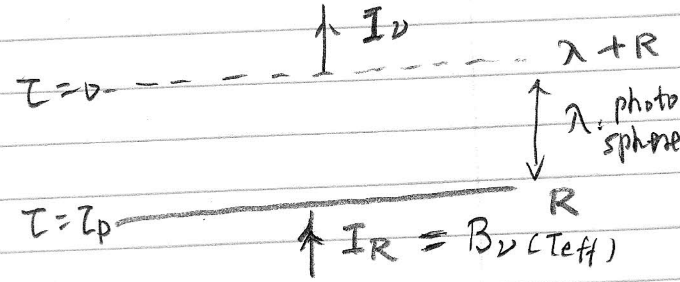
* Continuum blackbody radiation as from observations.

I_ν : line formation
Lorentz profile, and Voigt function

* Formation of Absorption Spectrum Lines

Ignore the radiation in the optical thin photosphere.

Only consider the radiation from the photospheric surface. and absorption in the photosphere



$$I_\nu = I_{R\nu} e^{-\tau_\nu}$$

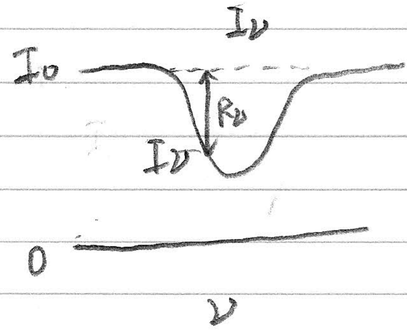
$$I_R = B_\nu(T_{eff})$$

$$\tau_\nu = \underbrace{\tau_e + \tau_{ff} + \tau_{bf}}_{\text{continuum spectrum}} + \underbrace{\tau_{bb}}_{\text{line spectrum}} = \tau_c + \tau_\ell$$

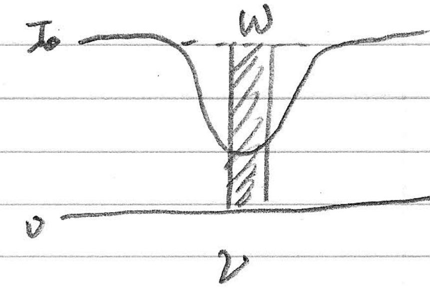
Because additional line absorption, radiation intensity is smaller in spectral lines than in continuum

Equivalent Width "w"

— a simple measure of the total absorption of line.



$\Rightarrow$



$$R_\nu = \frac{I_0 - I_\nu}{I_0} = 1 - \frac{I_\nu}{I_0}$$: normalized line depth at ν

$$w = \int_{Line} R_\nu \, d\nu = \int_{Line} \left(1 - \frac{I_\nu}{I_0}\right) d\nu$$

Equivalent width: Area of rectangle with width w and depth I_0 is the same as the total area of the line spectrum

w : small, weak line

w : large, strong absorption line