Structure and Evolution of the Sun (CH9)

part 1: An Introduction (see PPT presentation)

Part 2: Evolution from the ZAMS (CH9.2.2)

As a star evolves on the Main Sequence, the fundamental change is at the core:

$$\boxed{4H \longrightarrow He}$$ the nuclear burning

$\Rightarrow$ $\mu(t)$: μ as a function of time

mean molecular weight changes.

$X(H) \downarrow$, $Y(He) \uparrow$.

$$\boxed{\mu(t) = \frac{4}{3+5X(t)}} \quad \text{------} \quad (9.5)$$

Therefore, the change of $\mu(t)$ drives the evolution

For ZAMS, $t=0$, $X=0.7$, $\mu=0.6$

At the end of Main Sequence, $X=0$, $\mu=1.3$

* Let's prove $L \propto \mu^{7.5}$ --- (9.4)

— As μ increases, the luminosity increases

$$\frac{L}{L_0} = \left(\frac{1.3}{0.6}\right)^{7.5} \sim 400$$

This is the beginning of a red giant

From the diffusive radiation equation

$$F = \frac{4\pi}{3}\frac{\partial B}{\partial \tau}$$

$$L = -\frac{16\pi ac}{3\kappa\rho} r^2 T^3 \frac{dT}{dr}$$

Use Krammer's opacity $\kappa = K_0 \rho T^{-3.5}$ (K_{ff}, K_{bf})

$$\rho \propto \left(\frac{M}{R}\right)^{\frac{1}{3}}$$

$$\frac{dT}{dr} \sim \frac{T}{R}$$

The dimensional analysis

$$\Rightarrow L \propto \frac{1}{K_0 \rho T^{-3.5}} R^2 \cdot T^3 \frac{T}{R} \propto \frac{R T^{7.5}}{K_0 \rho} \quad \text{--- (9.2)}$$

Note. an error in (9.2) in the book, $T^4 \to T^{7.5}$

Use Virial Theorem, $U = -\frac{\Omega}{2}$

$$U = V \cdot E = \frac{M}{\rho} \cdot \frac{3}{2} n k T = \frac{M}{\rho} \cdot \frac{3}{2} \left(\frac{\rho}{\mu} N_A\right) kT$$

$$\Omega = -\frac{3}{5} \frac{GM^2}{R}$$

$$\Rightarrow T \propto \mu M^{\frac{2}{3}} \rho^{\frac{1}{3}} \quad \text{--- (9.1)}$$

plug into L in (9.2).

$$\Rightarrow L \propto \frac{M^{5.33} \rho^{0.017} \mu^{7.5}}{K_0} \quad \text{--- (9.3)}$$

Since M is a constant, mass loss $\sim 10^{-4} M$ during the Main Sequence

ρ: weakly dependent on ρ

$$\Rightarrow L \propto \mu^{7.5}$$

$$\Rightarrow \frac{L(t)}{L(0)} = \left[\frac{\mu(t)}{\mu(0)}\right]^{7.5} \quad \text{--- (9.4)}$$

* Find $L(t)$ as a function of t.

$$L(t) = -M \frac{dx}{dt} \cdot Q$$

$-M \frac{dx}{dt}$: mass reduction rate of hydrogen

Q : hydrogen burning energy release rate

$Q = 6 \times 10^{18}$ ergs g^{-1}

$$\frac{dX(t)}{dt} = -\frac{L(t)}{MQ} \quad \text{---- (9.6)}$$

$$\frac{d\mu(t)}{dt} = -\frac{5}{4}\mu(t)^2 \frac{dX(t)}{dt} = \frac{5}{4}\mu(t)^2 \frac{L(t)}{MQ} \quad \text{-- (9.7)}$$

From $\frac{d}{d\mu}$ of Eq (9.4)

$$\frac{1}{L_0}\frac{dL(t)}{d\mu} = 7.5 \frac{\mu^{6.5}}{\mu(0)^{7.5}} \frac{d\mu}{dt}$$

$$\Rightarrow \frac{dL(t)}{dt} = \frac{dL(t)}{d\mu} \cdot \frac{d\mu(t)}{dt}$$

$$\Rightarrow \frac{dL(t)}{dt} = \frac{75}{8}\frac{\mu(0)}{MQ}\frac{L(t)^{1+\frac{17}{15}}}{L(0)^{-1+\frac{17}{15}}} \quad \text{--- (9.8)}$$

$$\Rightarrow \boxed{L(t) = L(0)\left[1 - \frac{85}{8}\frac{\mu(0) L(0)}{MQ} t\right]^{-15/17}} \quad \text{--- 9.9}$$

For the Sun, $\mu(0) = 0.6$, $t_0 = 4.6 \times 10^9$ yrs,

$\Rightarrow L_\odot = 1.27 L(0)$

or $L(0) = 0.79 L_\odot$.

Thus, current Sun is about 27% brighter.

Further, current Sun is about 10% larger in radius