Nov. 13, 2012

CH5. Heat Transfer by Convection

Energy $\qquad$ $v \neq 0$

① Radiation: $v = 0$, high T

② conduction: $v = 0$, high ρ

③ convection: $v \neq 0$, high $\left|\frac{dT}{dr}\right|$

All three ways of energy transfer are proportional to $\boxed{\frac{dT}{dr}}$

Fick's Law of Diffusion: $F = -D \frac{d\phi}{dr}$

or $F = -D_T \frac{dT}{dr}$

① Radiation $F = -D \frac{du}{dr} = -D_T \frac{dT}{dr} = -\frac{4ac}{3\kappa\rho} T^3 \frac{dT}{dr}$

$D_{rad} = \frac{1}{3} c \lambda_{ph}$ $\qquad$ $D_T = \frac{4ac}{3} \frac{1}{\kappa\rho} T^3$

$u = aT^4$

② Conduction $F = -D \frac{dQ}{dr} = -D_T \frac{dT}{dr} = -\frac{1}{3} C_V v_e \lambda_e \frac{dT}{dr}$

$D_{cond} = \frac{1}{3} v_e \lambda_e$, $\quad Q = C_V T$

$D_T = \frac{1}{3} C_V v_e \lambda_e$

③ Convection $F = -D \frac{dQ}{dr}$

$F = \rho w C_p \Delta T$

ρ: density ; C_p: specific heat

w: average convection velocity

ΔT: temperature difference between the convection parcel and ambient

we need to find w and ΔT, given P, ρ, T [Q2]

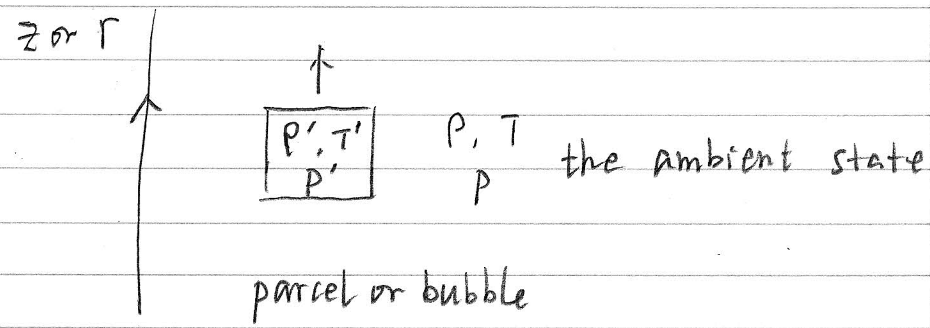
* In ZAMS, F_{rad} dominates

$F_{rad} \sim \frac{dT}{dr}$

However, $\left|\frac{dT}{dr}\right| > \left|\frac{dT}{dr}\right|_{critical} \Rightarrow$ Convection occurs [Q1]

Criteria for Convection (CH 5.1.1)

* Convection is caused by $\boxed{\text{high temperature gradient}}$
* The movement of convection parcel is driven by the $\boxed{\text{buoyancy force}}$



Consider a small disturbance of rise-up motion: w.

① the parcel is adiabatic (for now)

② the pressure internal is always in equilibrium with external : $P' = P$, since $w \ll C_s$ sound speed

$$P = nkT = \frac{\rho}{\mu} N_A kT$$

If $T' > T \Rightarrow \rho' < \rho$

The buoyancy force (Archimedes' principle)

$$F = \rho g - \rho' g = (\rho - \rho') g$$

If internal ρ' smaller, positive F, continue to rise

If " " larger, negative F, return to equilibrium (quenching the disturbance)

Thus, the criterion is

$$\left|\frac{dT'}{dz}\right| < \left|\frac{dT}{dz}\right|$$, large temperature gradient in ambient

Since $\frac{dT}{dz} < 0$, $\frac{dT'}{dz} > \frac{dT}{dz}$

Introduce β: $\boxed{\beta = -\frac{dT}{dz}}$ $\Rightarrow \beta' < \beta$

β (or $-\frac{dT}{dz}$) is determined by the stellar model

β' (or $-\frac{dT'}{dz}$) $= \beta_{ad}$, often consider an adiabatic process.

β_{ad} can be determined well because of "ad" constraints

$$\beta = -\frac{dT}{dz} = -\frac{dT}{dp}\frac{dp}{dz} = -\frac{T\frac{dT}{T}}{p\frac{dp}{p}} \cdot \frac{p\frac{dp}{p}}{dz}$$

$$\beta = -T\frac{d\ln T}{d\ln p}\frac{d\ln p}{dz}$$

Recall pressure scale height λ_p

$$\lambda_p = -\left(\frac{d\ln p}{dz}\right)^{-1} = \frac{P}{g\rho} = \frac{V_s^2}{g\Gamma_1}$$

$$P = P_0 e^{-\frac{z}{\lambda_p}}$$

$$\Gamma_1 = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{ad} = \frac{\rho}{P}\left(\frac{\partial P}{\partial \rho}\right)_{ad}$$

$$\left(\frac{\partial P}{\partial \rho}\right)_{ad} = V_s^2 . \quad V_s\text{: sound speed}$$

Recall the definition of $\left(\frac{d\ln T}{d\ln p}\right)_{ad}$

$$\boxed{\nabla = \frac{\Gamma_2 - 1}{\Gamma_2} = \left(\frac{d\ln T}{d\ln p}\right)_{ad}} \quad \text{--- } 3.94$$

∇ is Logarithm temperature gradient with respect to pressure.

Thus, $\beta = \frac{T}{\lambda_p}\nabla$.

\# "∇" a convenient dimensionless number

* For ideal gas, $\Gamma_2 = \frac{5}{3}$. $\nabla_{ad} = \frac{\frac{5}{3}-1}{5/3} = \frac{2}{5} = \underline{0.4}$

* For mixture (radiation plus ideal gas).

$$\nabla_{ad} = \frac{2(4-3\beta)}{32-24\beta-3\beta^2} . \quad \beta = \frac{P_g}{P_g + P_{rad}}$$

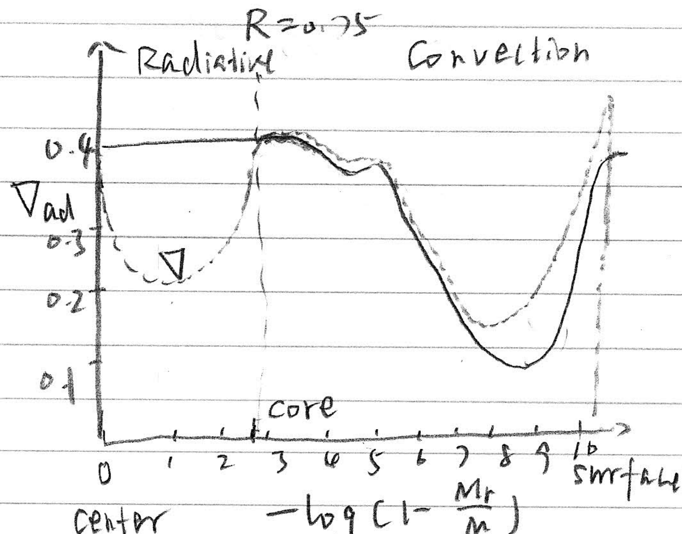
* For ionization ∇_{ad} is a function of Y. $\longrightarrow 0.1$

Thus, the criteria for convection

$$\left|\frac{dT}{dz}\right| > \left|\frac{dT'}{dz}\right|$$

$$\beta > \beta_{ad}$$

$$\nabla > \nabla_{ad}$$



Sun. Fig 5.2

x axis of $-\log(1-\frac{M_r}{M})$ emphasize the surface

$x=1$, $1-\frac{M_r}{M} = 10^{-1}$ $\Rightarrow$ $\frac{M_r}{M} = 1-10^{-1}$

or 10^{-1} (10%) mass outside

$1-10^{-1}$ (90% mass inside

$x=3$. $\frac{M_r}{M} = 1-10^{-3}$; 10^{-3} (0.1%) mass outside

99.9% mass inside

$x = 2.5$: separate the convection zone ($R \simeq 0.75$)

convection zone has a mass $< 1\%$

Mixing Length Theory (CH 5.1) — MLT

How to find F_{cond}?

How to find w and ΔT?

Use MLT to simplify the solution.

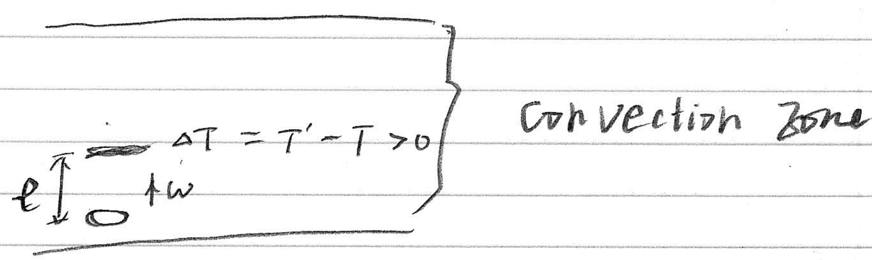
MLT: assume the convective parcel travels a characteristic distance "ℓ", the mixing length, before the parcel breaks up, lose identity and merge with the ambient fluid.

"ℓ" is usually a fraction of "λ_p"

To find w and ΔT, we need to look into

① the energy equation (CH 5.1.2)

② the equation of motion (CH 5.1.3)



Energy Equation (Radiative Leakage CH 5.1.2)

Since $v \neq 0$, $\frac{DA}{Dt} = \frac{\partial A}{\partial t} + \vec{W} \cdot \nabla A$: A any variable

1D $\frac{DT}{Dt} = \frac{dT}{dt} + w \frac{dT}{dz}$: $A = T$

Total derivative Lagrangian form — Local derivative Euler form — Advective (convective) component.

Thus, for ambient : $\frac{dT'}{dt} = 0$ (no local change)

$$\frac{DT}{Dt} = w \frac{dT}{dz} = -\beta w$$

For the parcel :

$$\frac{DT'}{Dt} = \frac{dT'}{dt} + w \frac{dT'}{dz} \quad \text{--- (5.18)}$$

For adiabatic convection process only,

$$\frac{DT'}{Dt} = -\beta_{ad}$$

$$\Rightarrow \frac{DT'}{Dt} = -\beta_{ad} w$$

$$\boxed{\frac{D(\Delta T)}{Dt} = (\beta - \beta_{ad}) w} \quad \text{--- 5.21}$$

$\Delta T = T' - T$

The energy equation in adiabatic case

* non-adiabatic : $\boxed{\frac{dT'}{dt} \neq 0}$ radiation from parcel to the surrounding is considered.

The parcel cools down due to radiation because of $T' > T$;
This is in addition to adiabatic expansion cooling.

* Radiative Leakage

$$\frac{dQ'}{dt} = -\nabla \cdot F_{rad} \quad \text{---} \quad (5.11)$$

$dQ' = C_p \rho \, dT'$, $\quad C_p$: due to pressure equilibrium

$$F_{rad} = -K \nabla T$$

$$K \equiv \frac{4acT^3}{3\kappa\rho}$$

$$\nabla \cdot (\nabla T) = \nabla^2 T$$. Laplace

$$\Rightarrow \left(\frac{\partial T'}{\partial t}\right) = \frac{4acT^3}{3\kappa\rho^2 C_p} \nabla^2 T \quad \text{------} \quad (5.15)$$

Introduce the thermal diffusivity ν_T

$$\boxed{\nu_T = \frac{4acT^3}{3\kappa\rho^2 C_p}} \quad \text{------} \quad (5.16)$$

$$\left(\frac{\partial T'}{\partial t}\right) = \nu_T \nabla^2 T \quad \text{--} \quad (5.17)$$

To find $\nabla^2 T$, we resort to mixing length

$$\nabla^2 T = \frac{T - T'}{l^2}$$

Add $\left(\frac{\partial T'}{\partial t}\right)$ term to Eq. (5.18)

$$\Rightarrow \frac{D\Delta T}{Dt} = (\beta - \beta_{ad}) w - \frac{\nu_T}{l^2} \Delta T \quad \text{---} \quad (5.21)$$

Assuming β, β_{ad}, ν_T, l^2 are known, find ΔT and w, need one more differential equation