Oct. 23, 2012 ①

CH 4: Radiative and Conductive Heat Transfer

* So far, we have worked out "easy" equations

① mass conservation $dM_r = 4\pi r^2 \rho(r)\, dr$

② momentum conservation $\frac{dP}{dr} = -\rho g$

(hydrostatic equilibrium)

③ equations of state $P = (\Gamma_3 - 1)\rho E$

Now, it is time to figure out the energy equation

④ $$\frac{dL}{dr} = 4\pi r^2 \rho(r)\, \varepsilon(r) \quad \text{(in nuclear core)}$$

$$\frac{dL}{dr} = 0 \quad \text{(outside the core)}$$

To fully address the energy equation, we need

⑤ $\varepsilon(\rho, T) = \varepsilon_0 \rho^\lambda T^\nu$ (nuclear physics) in CH6

⑥ $L = -D \frac{d(aT^4)}{dr}$ (energy transfer)

Three ways of transfer: D conduction (CH4)

D radiation transfer (CH4)

D convection (CH5)

Fick's Law of Diffusion: $F = -D\frac{d\phi}{dr}$, ϕ: concentration

D: diffusion coefficient

$$D_{rad} = \frac{c}{3\kappa\rho} = \frac{1}{3} c \lambda_{free} \quad \text{(CH4)}$$

⑦ $\kappa = \kappa_0 \rho^n T^{-s}$: opacity (CH4)

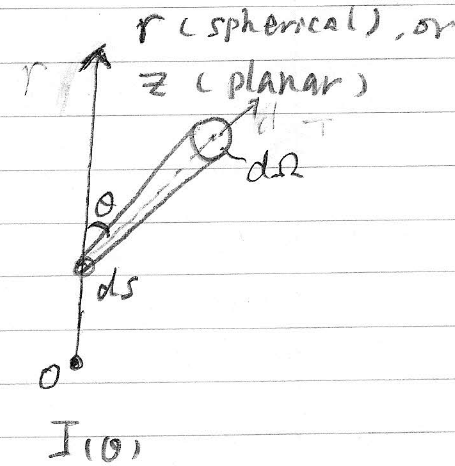
Goals ⇒ $T(r)$, $\rho(r)$, $P(r)$, $M(r)$, $L(r)$, $\varepsilon(r)$, $\kappa(r)$

4.1 Radiative Transfer.

The goal is to find $L = -D_{rad} \frac{d(aT^4)}{dr}$

what is D_{rad}?

* Specific Intensity $I(\theta)$.



Radiation energy passing through a unit area (ds) along a specific direction (θ, colatitude) in a unit time (dt) and within a unit solid angle ($d\Omega$)

$$I(\theta) = \frac{dE}{ds \cdot dt \cdot d\Omega} \quad erg\ cm^{-2}\ s^{-1}\ sr^{-1}$$

Sr: steradian, 4π for a sphere

$d\Omega = \sin\theta\, d\theta\, d\varphi$ $\quad (dA = r^2 d\Omega) = (r d\theta \cdot r\sin\theta\, d\varphi)$

$(A = 4\pi r^2)$

* $\mu = \cos\theta$

θ: colatitude

φ: azimuthal angle

$d\Omega = d\mu\, d\varphi$

$$\Omega = \int_{-1}^{1}\int_{0}^{2\pi} d\mu\, d\varphi = 2 \cdot 2\pi = 4\pi$$

$I(\theta < \frac{\pi}{2}) = I(\mu > 0)$: outgoing radiation

$I(\theta > \frac{\pi}{2}) = I(\mu < 0)$: inward radiation

* Some radiation parameters:

u: energy density

F: radiation flux

* radiation energy density u

$$I(\theta)\,d\Omega = \frac{dE}{dt\,ds} = erg\ cm^{-2}\ s^{-1}$$

$\equiv$ energy in cone $d\Omega$ in a volume : dV

$$dV = ds.\,dl = ds.\,(c\,dt)$$

Thus the energy density in cone $d\Omega$

$$du = \frac{dE}{dV} = \frac{I(\theta)\,d\Omega.\,dt.\,ds}{c\ ds\ dt} = \boxed{\frac{I(\theta)}{c}\,d\Omega} \quad \text{-- (4.1)}$$

Total energy density (erg cm^{-3})

$$u = \int du = \int \frac{I(\theta)}{c}\,d\Omega = \frac{2\pi}{c}\int_{-1}^{+1} I(\mu)\,d\mu \quad \text{... (4.2)}$$

For LTE and complete isotropic $I(\mu) = I$

$$\boxed{u = \frac{2\pi}{c}.\int_{-1}^{+1} I\,d\mu = \frac{4\pi}{c}\,I} \quad \text{---- (4.5)}$$

In LTE : $u = aT^4$

$$\Rightarrow I = \frac{c}{4\pi}\,aT^4 = \frac{\sigma T^4}{\pi} = B(T) \quad \text{-- (4.6)}$$

$\sigma = \frac{ac}{4}$: stephan – Boltzmann constant

$= 5.67 \times 10^{-5}$ erg $cm^{-2}\ K^{-4}\ s^{-1}$

a : radiation constant $= 7.566 \times 10^{-15}$ erg $cm^{-3}\ K^{-4}$

$a = \left(\frac{k^4}{c^3 h^3}\cdot\frac{8\pi^5}{15}\right)$ from distribution function

$B(T)$ = integrated planck function

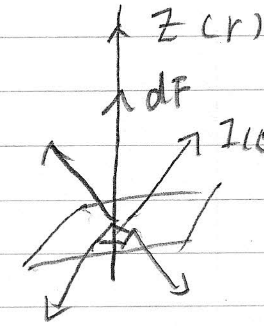
$$B = \int B(\nu)\,d\nu$$

(planck function:) $B(\nu) = \frac{2h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kT} - 1}$ erg $cm^{-2}\ s^{-1}\ sr^{-1}\ Hz^{-1}$

* radiation flux F: the net flux, or total flux

Flux contributed by $I(\theta)\,d\Omega$

$dF = \cos\theta\, I(\theta)\, d\Omega$



$\cos\theta$: geometric projection effect

$F = \int dF = \int_{4\pi} I(\theta) \cos\theta\, d\Omega$

$\equiv$ total radiation flux across the surface along the r (or z) direction

$F = 2\pi \int_{-1}^{+1} I(\mu)\, \mu\, d\mu \quad erg\ s^{-1}\ cm^{-2}$ ... (4.3)

In LTE, perfect black body, complete isotropic $I(\mu) = B(T)$

$F = 2\pi B \int_{-1}^{+1} \mu\, d\mu = 2\pi B \cdot \frac{1}{2}\mu^2\Big|_{-1}^{+1} = 0$

* net flux $F = 0$ for isotropic radiation

* For a star $L = 4\pi r^2 F$

$L \neq 0$, $F \neq 0$.

$\Rightarrow$ radiation is not complete isotropic

$\Rightarrow$ slight anisotropic

$\approx 10^{-9} B$

or $I_{out} \approx I_{in} \approx B$ tiny deviation from isotropic

but $I_{out} - I_{in} \approx 10^{-9} B$.