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1.4.1. Molecular Weight μ ($n = \frac{\rho}{\mu} N_A$)

μ: mass per particle in a mixture, unit (amu)

For pure hydrogen atom, $\mu_H = 1$ (amu)

amu: atomic mass unit; ^{12}C exactly equal 12 amu

$1 \text{ amu} = 1.6605 \times 10^{-24}$ g

$\mu_H = 1.00794$ amu

$\mu_{He} = 4.002602$ amu

$$\frac{4\mu_{He}}{\mu_{He}} = 1.00728$$, or 0.7% more massive

$N_A = 6.022 \times 10^{23}$ mole^{-1} (Avogadro's number)

$N_A = \frac{1}{1\text{ amu}(g)}$: number of ^{12}C in 12 grams

For a mixture: neutral atoms, ions, electrons with different species (H, He, metals)

what is mean molecular weigh μ?

* mean molecular weight of ions: μ_I

ion: include neutral atoms and ions

X_i: mass fraction of ion i

$\sum X_i = 1$.

$X_H \approx 0.7$

$X_{He} \approx 0.3$

$X_{metal} \approx 0.02$ for the Sun

$$n_I = \frac{\rho}{\mu_I} N_A$$

$$n_I = \sum_i n_{I,i}$$

$$n_{I,i} = \frac{\rho X_i}{A_i} N_A \qquad \text{—— 1.43}$$

A_i : atomic mass number of ion i

$$\Rightarrow n_I = \rho N_A \sum_i \frac{X_i}{A_i} \qquad \text{—— 1.44}$$

$$\Rightarrow \mu_I = \left[\sum_i \frac{X_i}{A_i} \right]^{-1} \qquad \text{---- 1.46}$$

For the Sun: $X_H = 0.7$, $A_H = 1$

$X_{He} = 0.3$ $A_{He} = 4$

$X_{metals} = 0.02$ $A_{metal} \approx 14$ (average)

$$\mu_I = \left[\frac{0.7}{1} + \frac{0.3}{4} + \frac{0.02}{14} \right]^{-1} \approx 1.3$$

* Mean molecular weight per electron: μ_e

$$n_e = \frac{\rho}{\mu_e} N_A \qquad n_e \text{: electron density}$$

$$n_{e,i} = n_{I,i} \cdot y_i \cdot Z_i$$

y_i : ionization fraction $\begin{cases} y = 0, & \text{neutral} \\ y = 1 & \text{fully ionized} \end{cases}$

Z_i : charge number

$$n_{e,i} = \rho N_A \left(\frac{X_i}{A_i} \right) y_i Z_i$$

$$n_e = \sum_i n_{e,i} = \rho N_A \sum_i \left(\frac{X_i}{A_i} \right) y_i Z_i$$

$$\Rightarrow \mu_e = \left[\sum_i \frac{Z_i X_i y_i}{A_i} \right]^{-1} \qquad \text{---- 1.49}$$

* Total mean molecular weight μ

$$n = n_I + n_e = \frac{\rho}{\mu} N_A$$

$$n = \frac{\rho}{\mu_I} N_A + \frac{\rho}{\mu_e} N_A = \frac{\rho}{\mu} N_A$$

$$\Rightarrow \frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_e}$$

$$\mu = \left[\frac{1}{\mu_I} + \frac{1}{\mu_e} \right]^{-1} \quad \cdots\cdots \quad (1.52)$$

For the Sun.

$\mu_I \approx 1.3$, $\mu_e \approx 1.2$, $\mu \approx 0.6$