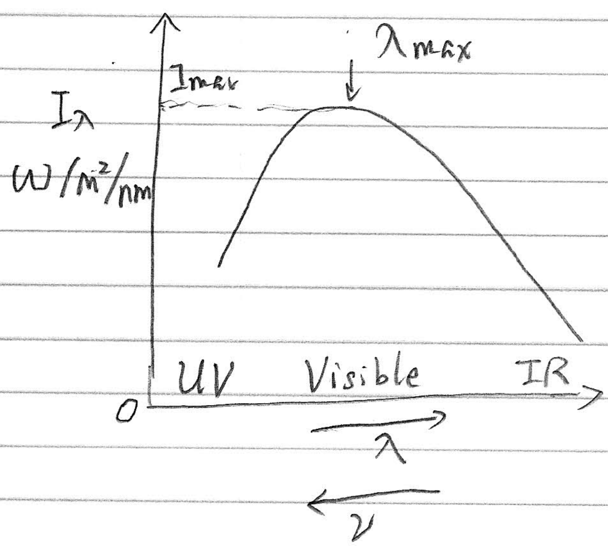
2011 May 4

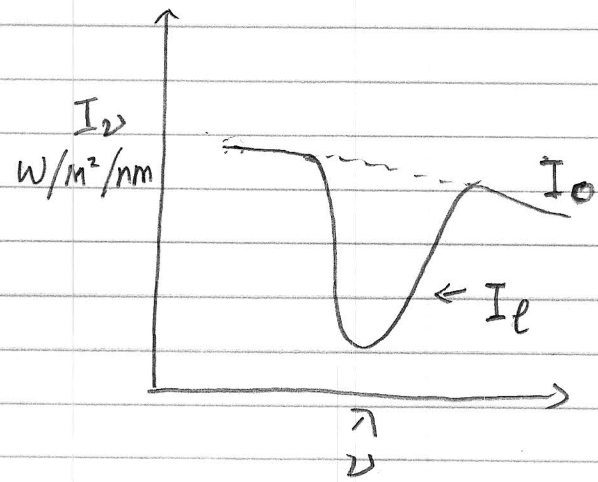
Stellar Atmosphere — Line Profiles and Curve of Growth
— Equivalent Width

CH 4.8 + this note

* Stellar spectrum



Low Resolution
Continuum spectrum

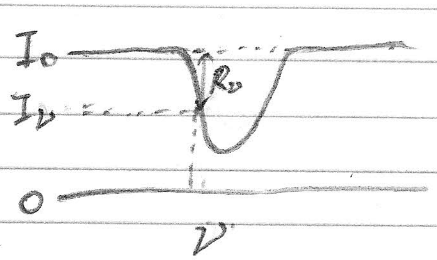


high resolution
Continuum + absorption Line

$I_\nu = I_0 + I_\ell$, $I_\ell < 0$ (absorption)

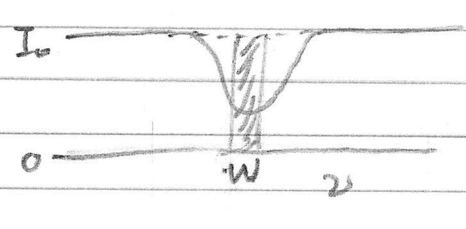
$I_\ell > 0$ (emission)

* Equivalent width "W":
a simple measure of the total absorption of the line



$$R_\nu = \frac{I_0 - I_\nu}{I_0} = 1 - \frac{I_\nu}{I_0}$$

R_ν : line depth at ν



$$W = \int_{Line} R_\nu \, d\nu = \int_{Line} \left(1 - \frac{I_\nu}{I_0}\right) d\nu$$

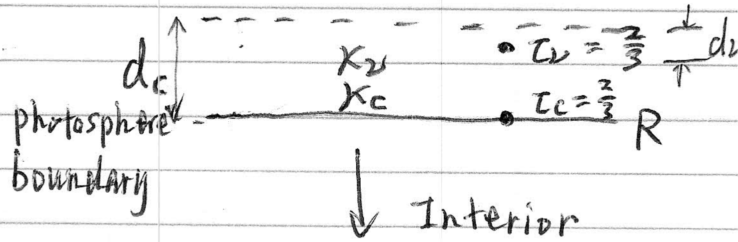
Area of rectangle W is the same as the total area of the line

* Formation of spectral Lines

Continuum $I_0 = B(\tau_c = \frac{2}{3})$, $B = \frac{\sigma T^4}{\pi}$

$$\tau_c = K_c \rho d = \frac{2}{3}$$

K_c: mass absorption coefficient due to continuum mechanism.



$K_c = K_e + K_{ff} + K_{bf}$

K_e: Thomson scattering due to free electrons

K_{ff}: free-free absorption due to free electrons interacting with ions

K_{bf}: bound-free absorption

Absorption Line $I_\nu = B(\tau_\nu = \frac{2}{3})$

$$\tau_\nu = K_\nu \rho d = \frac{2}{3}$$

$K_\nu = K_c + K_{bb}$ = continuum absorption + bound-bound absorption

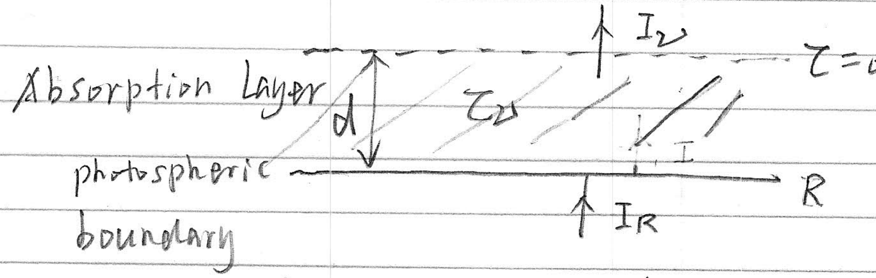
$K_\nu > K_c$

$$d_\nu = \frac{\tau_\nu}{K_\nu \rho} < \frac{\tau_\nu}{K_c \rho}$$

$d_\nu < d_c$ $\Rightarrow$ photons come from a shallower layer in spectral lines than that in continuum.

In shallower layer, Temperature lower, $\Rightarrow I_\nu < I_0$
$\Rightarrow$ lower radiation Intensity

Formation of Spectral Line — Alternative Way



Emission layer at $r=R$, the photospheric boundary.
The atmosphere is an absorption layer, and ignore the radiative transfer emission component

$$I_\nu = I_R e^{-\tau_\nu} \quad , \quad \tau_\nu = \kappa_\nu \rho d$$

$$\tau_\nu > \tau_c \Rightarrow I_\nu < I_c$$

At continuum I_c, $\tau_\nu = \frac{2}{3}$

At spectral lines, $\tau_\nu > \frac{2}{3}$

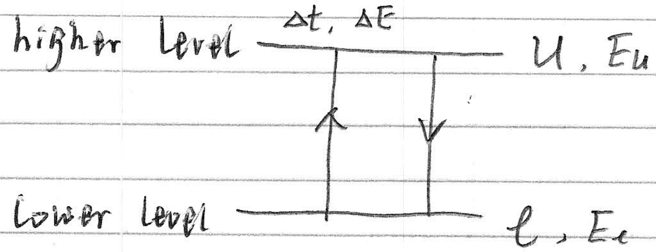
$$\kappa_\nu > \kappa_c \Rightarrow \tau_\nu > \tau_c \Rightarrow I_\nu < I_c$$

If κ_ν very large, $\tau_\nu >> 1 \Rightarrow I_\nu = 0$

saturated absorption

Line profiles — Lorentz Profile (CH 4.8.1)

spectral line is caused by bound-bound transition between different energy levels



Transition between Atomic Levels

$h\nu_0 = E_u - E_l$

ν_0: photon frequency

However, the life time for the transition is finite

$\tau = 1.6 \times 10^{-9}$ s for Hydrogen Lyman α (2P–1S)

It is not infinite ~~small~~ long $\tau \to \infty$

* Heisenberg uncertainty principle

$\Delta E \, \Delta t = \hbar$; $\hbar = \frac{h}{2\pi}$ $h = 6.626 \times 10^{-27}$ erg s planck's constant

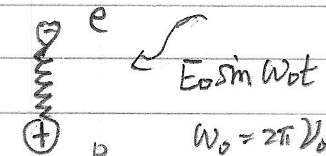
ΔE is finite $\Rightarrow$ $\Delta\nu$

$\Delta \nu = \frac{\Delta E}{h}$

* Spectral line has a natural width $\Delta\nu$

profile of the width : $\boxed{\text{classical oscillator}}$ approach

The oscillator undergoes an stimulated oscillation, for given external electric field $E_0 \sin \omega_0 t$, $\omega_0 = 2\pi\nu_0$. caused by incident photons



Atom is treated as a classical oscillator

Equation of motion of bound electron

$$m_e \frac{d^2 z}{dt^2} + \gamma \frac{dz}{dt} + \omega_0^2 z = \frac{e}{m} E_0 e^{i\omega t}$$

$z = z_0 e^{i\omega t}$, γ : damping constant due to radiation

(continued)

$$z = \frac{e}{m_e}\frac{E_0}{w_0^2 - w^2 + i\gamma w}$$; amplitude of the oscillator

$w = w_0$: z highest. w_0: resonant frequency

$$I_w = |E|^2$$

$w \neq w_0$: z smaller.

Ignore the derivation here. but you gets the sense of the damping profile (or Lorentz profile)

$$\sigma_a = \frac{e^2}{mc} f \frac{(\gamma/4\pi)}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2} \quad \text{------} \quad (4.73)$$

σ_a: absorption cross section

$\sigma_a \cdot n_a = \kappa\rho$; n_a: number of absorbers

f: oscillation strength

$$f = \frac{mc^3}{2\pi e^2 \nu^2}\frac{g_u}{g_l} A_{ul}$$ from quantum mechanics

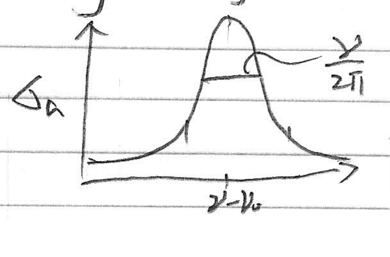
A_{ul}: Einstein probability coefficient

g_u, g_l: statistical weight of upper, low levels

$$\gamma_u = 4\pi \sum_{l<u} A_{ul}$$

$$\gamma = \gamma_u + \gamma_l$$

γ determine the natural width of spectral lines but usually very small. $\Delta\lambda \sim 10^{-4}$ A



$\Leftarrow$ Lorentz profile

Full width at half maximum $= \frac{\gamma}{2\pi}$

$\boxed{\text{narrow core + broad wing}}$

\# Doppler Broadening (CH 4.8.2) — thermal broadening

the natural width is narrow $\sim 10^{-4}$ Å.

the true width of spectral line is mainly caused by Doppler broading, since Temp $\sim 10^3 - 10^4$ K in stellar atmosphere

Doppler shift: $\Delta\nu = \frac{v}{c}\nu$; $\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} = \frac{v}{c}$

v : velocity of atoms

$v_0^2 = \frac{2kT}{m} \Rightarrow \boxed{\Delta\nu_0 = \frac{v_0}{c}\nu_0}$ Doppler width

$$\frac{dn(v,\mu)}{n} = (2\pi)^{-\frac{1}{2}}\left(\frac{m}{kT}\right)^{\frac{3}{2}} e^{-mv^2/kT}\, v^2\, dv\, d\mu$$

Maxwell-Boltzmann distribution of thermal plasma/gas

One need to convolve this velocity profile to Lorentz profile

$$\sigma_\nu(\nu, \nu_0, T) = \frac{1}{(2\pi)^{\frac{1}{2}}}\left(\frac{m}{kT}\right)^{\frac{3}{2}} \frac{c^2}{mc} f \frac{\gamma}{4\pi} \int_0^\infty \int_{-1}^{1} \frac{e^{-\frac{mv^2}{2kT}} v^2\, dv\, d\mu}{(\nu - \nu_0 - \nu_0 v\mu/c)^2 + (\frac{\gamma}{4\pi})^2}$$

$$\sigma_\nu = \frac{e^2 f}{mc} \pi^{\frac{1}{2}} \frac{1}{\Delta\nu_0} H(a, \Delta\nu/\Delta\nu_0) \quad \text{--- } 4.77$$

$$\Delta\nu_D = \frac{v_0}{c}\nu_0 = \nu_0\left(\frac{2kT}{mc^2}\right)^{\frac{1}{2}} \quad \text{--- } (4.78)$$

Doppler width

$a = \frac{\gamma}{4\pi}\frac{1}{\Delta\nu_D}$ ratio of natural width and Doppler width

$a = 10^{-3}$, the typical value

$\Delta\nu = \nu - \nu_0$

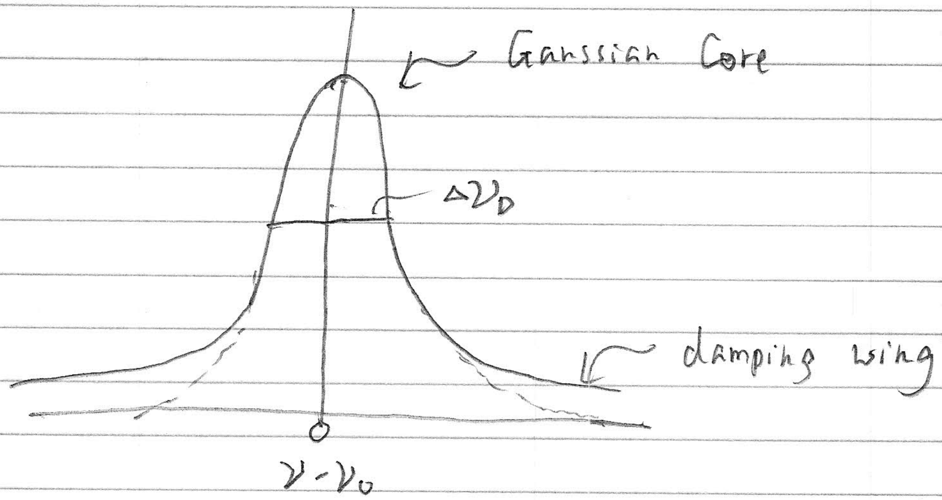
(Continued)

$$H(a, u = \frac{\Delta\nu}{\Delta\nu_D}) = \frac{a}{\pi}\int_{-\infty}^{+\infty} \frac{e^{-y^2}dy}{a^2 + (u-y)^2} \quad \text{--- (4.81)}$$

\# $\boxed{\text{The Voigt function}}$

$u \sim 1$, $H \sim e^{-u^2}$. Gaussian Core

$u \gg 1$, $H \sim \frac{1}{u^2}$: power-law wing; daming wing



Curve of Growth (CH 4.8-3)

The evolution of an absorption line with increasing number density of absorbers.

$$\frac{I_\nu}{I_0} = e^{-\tau_\nu} = e^{-\beta_0} = e^{-n_a \sigma_{a,\nu} \Delta r} \quad \text{--- (4.82)}$$

$\beta \ll 1$, weak line

$\beta \uparrow$, stronger absorption

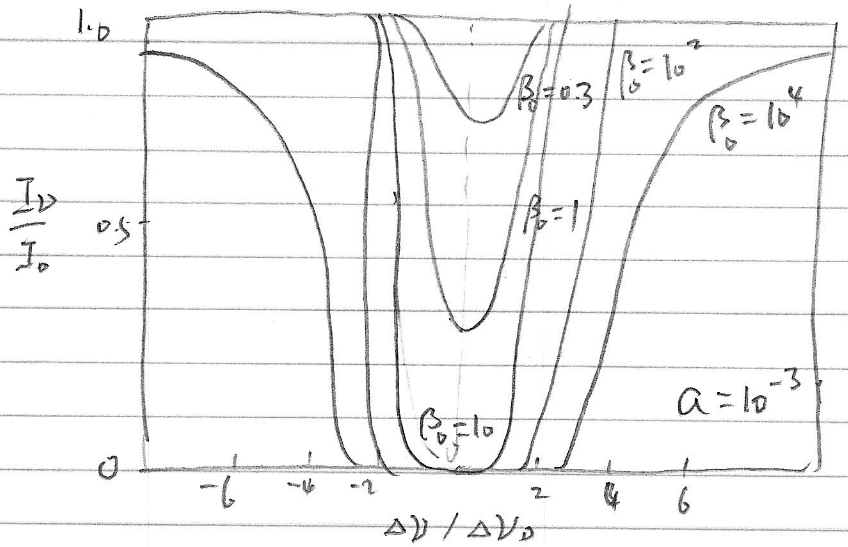
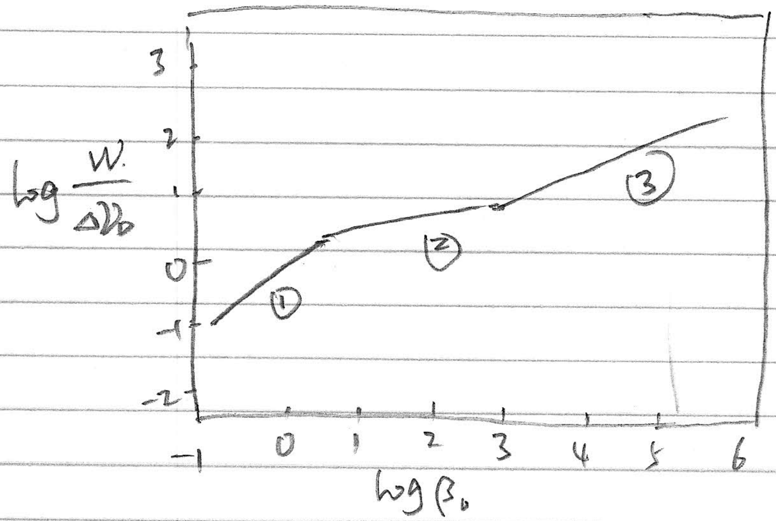


Figure 4.10 (P230)

Saturation: $\beta \sim 10$. all photons are absorped near the core of the Doppler width

(Continued)



Used to determine accurately
(1) temperature
(3) abundance

Fig 4.11 (P231) Curve of Growth

* Curve of Growth: how the line width changes with the number of absorbers, or τ_ν, or β_0

The curve has three segments

(1) Linear region. $W \sim \beta_0 \sim \tau$

$-1 < \log \beta_0 < 0$, $\tau < 1$, optical thin

weak lines,

the width $< \Delta\nu_0$, but grow linearly with τ

(2) Saturation region: ~~$W \sim \beta_0 \sim \tau$~~ $W \sim \sqrt{\ln \beta_0}$

$W \sim \Delta\nu_0$

$1 < \tau < 1000$ optical thick $(0 < \log\beta < 3)$

Gaussian core is saturated,

the line width $\sim$ Doppler width

(3) Damping region $W \sim \beta_0^{\frac{1}{2}} \sim \tau^{\frac{1}{2}}$

$\log \beta > 3$, $\tau > 1000$

Damping wing participates in the absorption

The line grows again, but not as steep as linear region