2011 April 27

Structure and Evolution of the Sun (CH 9)

$P(r)$, $\rho(r)$, $T(r)$, $L(r)$, $\varepsilon(r)$

Core: $0 < r < 0.3R$: radiative + $\varepsilon > 0$

Radiative zone, $0.3 < r < 0.75R$: radiative + $\varepsilon = 0$

Convection zone: $0.75R < r < 1.0R$: convective + $\varepsilon = 0$

Evolution from ZAMS: (CH 9.2.2)

Use dimensional analysis, to prove

$$L \propto \mu^{7.5}$$

$$P = nkT = \frac{N_A k}{\mu} \rho T.$$

As the Sun evolves, nuclear fusion H → He,

→ μ increases.

$$X + Y = 1 \Rightarrow \mu = \frac{4}{3+5X}$$

Initially, $X = 0.7$, $\mu = 0.6$

At the end of main sequence, $X = 0$, $\mu = 1.3$

$$\frac{L}{L_0} = \left(\frac{1.3}{0.6}\right)^{7.5} \sim 400.$$

This is the beginning of red giant.

From radiation diffusion: $F = \frac{4\pi}{3} \frac{\partial B}{\partial \tau}$

$$L_r = -\frac{16\pi ac}{3\kappa\rho} r^2 T^3 \frac{dT}{dr}$$

$$\frac{dT}{dr} \sim \frac{T}{R}, \quad \kappa = K_0 \rho T^{-3.5}, \quad \rho \propto \left(\frac{M}{R}\right)^{\frac{1}{3}}$$

$$\Rightarrow L \propto \frac{RT^4}{\kappa\rho} \qquad (9.2)$$

From Virial Theorem

$$U = -\frac{\Omega}{2}$$

$$U = V \cdot E = \frac{M}{\rho}\frac{3}{2}nKT = \frac{M}{\rho}\cdot\frac{3}{2}\frac{N_A K}{\mu}\rho T$$

$$\Omega = -\frac{3}{5}\frac{GM^2}{R}$$

$$\Rightarrow T \propto \mu M^{\frac{2}{3}}\rho^{\frac{1}{3}} \qquad (A.1)$$

$$\Rightarrow L \propto \frac{M^{5.33}\rho^{0.117}\mu^{7.5}}{K_0} \qquad \text{---} \; (A.3)$$

Since mass M does not change. and L weakly depends on ρ

$$\Rightarrow L \propto \mu^{7.5}$$

or $$\frac{L(t)}{L(0)} = \left[\frac{\mu(t)}{\mu(0)}\right]^{7.5} \qquad \text{- - - -} \; (A.4)$$

* $L(t)$? From ZAMS to "t" for the Sun

Further $L(t) = -\underbrace{M\frac{dX}{dt}}_{\text{Mass reduction rate of Hydrogen}} \cdot Q$

Q: hydrogen burning release rate

$Q = 6 \times 10^{18}$ ergs g^{-1}

$$\frac{dX(t)}{dt} = -\frac{L(t)}{MQ} \qquad \text{- - - - -} \; (A.6)$$

$$\frac{d\mu(t)}{dt} = -\frac{5}{4}\mu^2_{(t)}\frac{dX}{dt} = \frac{5}{4}\mu^2_{(t)}\frac{L(t)}{MQ} \qquad \text{--} \; (A.7)$$

Continued

$\frac{d}{dM}$ (Eq 9.4). $$\frac{1}{L_\odot}\frac{dL(t)}{dM} = 7.5\,\frac{M(t)^{6.5}}{M(0)^{7.5}}\,\frac{dM(t)}{dt}$$

and use (9.7) and (9.4) remove $M(t)$ and $\frac{dM}{dt}$

$$\Rightarrow \frac{dL(t)}{dt} = \frac{75}{8}\,\frac{M(0)}{MQ}\,\frac{L(t)^{1+\frac{17}{15}}}{L(0)^{-1+\frac{17}{15}}}$$

$$\Rightarrow L(t) = L(0)\left[1-\frac{85}{8}\,\frac{M(0)L(0)}{MR}\,t\right]^{-15/17} \quad (9.9)$$

$$\Rightarrow \frac{L(t)}{L_\odot} = \frac{L(0)}{L_\odot}\left[1-0.3\,\frac{L(0)}{L_\odot}\cdot\frac{t}{t_\odot}\right]^{-15/17} \quad \cdots (9.10)$$

where $t_\odot = 4.6\times10^9$ yrs. $L_\odot = 3.842\times10^{33}$ ergs s^{-1}

At $t=0$. ZAMS sun

$L(0) = 0.79\,L_\odot$.

Thus, current Sun is about 25% brighter.

Further, current Sun is about 10% larger in radius

Stellar Atmosphere (CH 4)

* CH 4.3 A simple Atmosphere : gray, independent of λ
* CH 4.8. Line profiles : dependent of λ

λ : wavelength, $\nu = \frac{c}{\lambda}$: frequency

A simple Atmosphere

show $T^4(\tau) = \frac{1}{2} T_{eff}^4 \left(1 + \frac{3}{2}\tau\right)$ ---- 4.44

* T_{eff}: effective temperature
* $\tau = 0$, true surface. $\rho = 0$, but $T \neq 0$, $P \neq 0$

$P = P_{rad} + P_{gas}$

what is T and P at $\tau = 0$, the true surface.

* Definition of R: the photosphere.

$$L = 4\pi \underline{R}^2 \sigma \underline{T_{eff}^4}$$

Above R, photosphere : optical thin, non-blackbody

Below R : optical thick, non-transparent, blackbody

* Using radiation pressure P_r

In LTE, $P_r = \frac{aT^4}{3}$ dyne cm^{-2}

$U_r = aT^4$ erg cm^{-3} (energy density)

$I = B = \frac{\sigma T^4}{\pi}$

$U_r = \frac{2\pi}{c} \int_{-1}^{+1} I(\mu)\, d\mu$ ---- (4.21)

$U_r = \frac{4\sigma}{c} T^4 = aT^4$; $a = \frac{4\sigma}{c}$

$P_r = \frac{2\pi}{c} \int_{-1}^{+1} I(\mu)\, \mu^2\, d\mu = \frac{1}{3} \cdot \frac{4\sigma}{c} T^4 = \frac{a}{3} T^4$

$P_r = \frac{4\pi}{3c} B$ ------ (4.32)

$F = \frac{4\pi}{3} \frac{\partial B}{\partial \tau} = \frac{L}{4\pi r^2}$ ------ (4.31)

$$\partial \tau = -\kappa \rho \, \partial r$$

$$\Rightarrow \frac{dP_r}{dr} = -\frac{\kappa \rho L}{4\pi r^2 c} \quad ; \quad \frac{dP_r}{d\tau} = \frac{L}{4\pi r^2 c}$$

For atmosphere (not interior). $r \approx R$.

$$P_r \Big|_0^\tau = \frac{L \, \tau}{4\pi R^2 c} \Big|_0^\tau$$

$L = 4\pi R^2 \sigma T_{eff}^4$. replace R with T_{eff}

$$P_r(\tau) = \frac{\sigma T_{eff}^4}{c} \tau + P_r(\tau = 0) \quad \text{------ (4.38)}$$

* $P_r(\tau=0)$? Radiation pressure at true surface.

Assuming isotropic radiation, but only outgoing

$$P_r(\tau=0) = \frac{2\pi}{c} \int_0^1 I(\tau=0) \, \mu^2 d\mu = \frac{2\pi}{3c} I(\tau=0) \quad \text{— (4.40)}$$

$$L = 4\pi R^2 \cdot 2\pi \int_0^1 I(\tau=0) \, \mu \, d\mu = 4\pi R^2 \pi I(\tau=0) \quad (4.41)$$

$$\Rightarrow P_r(\tau=0) = \frac{2\pi}{3c} \cdot \frac{L}{4\pi R^2} \frac{1}{\pi} = \frac{2}{3} \frac{\sigma T_{eff}^4}{c}$$

$$P_r(\tau) = \frac{1}{3} a T^4(\tau) \approx \frac{\sigma T_{eff}^4}{c} \left(\tau + \frac{2}{3} \right)$$

$$\Rightarrow T^4(\tau) = \frac{1}{2} T_{eff}^4 \left(1 + \frac{3}{2} \tau \right) \quad \text{------ (4.44)}$$

since $a = \frac{4\sigma}{c}$

* $\tau = \frac{2}{3}$, $T = T_{eff}$, $r = R$

photosphere at $\tau = \frac{2}{3}$

For the Sun, $\ell \approx 50$ km at $\tau = \frac{2}{3}$

* $\tau = 0$ true surface

$$T = 2^{-\frac{1}{4}} T_{eff} = 0.84 \, T_{eff} \neq 0$$

$$P_r(\tau=0) = \frac{2}{3} \frac{\sigma T_{eff}^4}{c} \neq 0$$