2011 April 6

Eddington Critical Luminosity, or Eddington Limit

$$F = \frac{4\pi}{3}\frac{\partial B}{\partial \tau} \qquad \text{--- (4.1a)}$$

$$F = -\frac{4ac}{3}\frac{1}{\kappa\rho}T^3\frac{dT}{dr} = -\frac{c}{\kappa\rho}\frac{d(\frac{1}{3}aT^4)}{dr}$$

$$P_{rad} = \frac{1}{3}aT^4$$

$$F = -\frac{c}{\kappa\rho}\frac{dP_{rad}}{dr}$$

$$L = 4\pi R^2 F = -\frac{4\pi R^2 c}{\kappa\rho}\frac{dP_{rad}}{dr} \qquad \text{--- (4.51)}$$

Eddington Limit: ~~grav~~ radiation pressure force exceeds gravitational force

$$\boxed{-\frac{dP_{rad}}{dr} = g_s\rho} \qquad \text{--- (4.50)}$$

$$L_{Edd} = \frac{4\pi \cancel{R^2} c}{\kappa \cancel{\rho}}\cdot g_s \cancel{\rho}\cdot \frac{GM}{R^2}$$

$$g_s = \frac{GM}{R^2}$$

$$\boxed{L_{Edd} = \frac{4\pi c\, G\, M}{\kappa}} \qquad \text{--- (4.52)}$$

Use $\kappa = \kappa_e = 0.2(1+X) = \underline{0.34}\ cm^2\, g^{-1}$

Assuming $X = 0.7$

$$\left(\frac{L_{Edd}}{L_\odot}\right) = 3.5\times10^4\left(\frac{M}{M_\odot}\right) \qquad \text{--- (4.52)}$$

For current Sun, radiation pressure is low.

However, in supergiants, radiation pressure drives strong stellar wind → mass loss → drive planetary nebula

Radiative Opacity Source (CH 4.4)

κ : determined by microphysics, function of ρ, T, composition

κ : $cm^2 g^{-1}$, $\kappa\rho = \frac{1}{\lambda_{free}}$

* $\kappa_{tot} = \kappa_e + \kappa_{ff} + \kappa_{bf} + \kappa_{H^-}$

κ_e : electron scattering — High T

κ_{ff} : free - free absorption

κ_{bf} : bound - free absorption

κ_{H^-} : metal hydrogen absorption — Low T

κ_{bb} : bound - bound absorption → spectral lines

* Electron scattering κ_e

In classic EM class, the so-called Thomson Electron scattering is caused by oscillation of free electrons upon the incidence of electro-magnetic radiation

$$\sigma_e = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = \frac{8\pi}{3} r_e^2 = 0.6652 \times 10^{-24} \, cm^{-2} \quad \text{-- (4.59)}$$

$$r_e = \frac{e^2}{m_e c^2} \; : \text{ classical electron radius } \left(\frac{e^2}{r_e} = m_e c^2\right)$$

total electronic energy (Coulomb) = rest mass energy

σ_e : cm^2 per particle

Cross-section (cm^2) indicates the probability of interaction

$\sigma_e n_e$: cm^2 / cm^3, probability of interaction unit volume

$\lambda \sigma_e n_e = 1$; $\lambda = \frac{1}{\sigma_e n_e}$

λ : definition of mean free path of photons

when photon moves a distance of λ, there is one interaction

Therefore $\lambda = \frac{1}{\sigma_e n_e} = \frac{1}{\kappa_e \rho}$

$$\kappa_e = \frac{\sigma_e n_e}{\rho}$$

For the matter of X fraction of H ($A=1$, $Z=1$), and other matters $1-X$ ($A=2Z$)

$$\mu_e = \left(\sum_i \frac{X_i Y_i Z_i}{A_i}\right)^{-1} = \left(\frac{X \cdot 1 \cdot 1}{1} + \frac{(1-X) \cdot 1 \cdot Z}{2Z}\right)^{-1}$$

$$\mu_e = \left(\frac{X+1}{2}\right)^{-1} = \frac{2}{X+1}$$

$$n_e = \frac{\rho}{\mu_e} N_A \Rightarrow \frac{n_e}{\rho} = \frac{N_A}{\mu_e}$$

$$\boxed{\kappa_e = \frac{\sigma_e N_A}{\mu_e} = 0.2(1+X)} \quad cm^2\, g^{-1} \quad \text{-- (4.60)}$$

Exp.: In the core of Sun, $X=0.7$, $\rho = 150\ g\ cm^{-3}$

$$\kappa_e \rho = 0.2 \cdot (1+0.7) \cdot 150 = 51\ cm^{-1}$$

$$\lambda = \frac{1}{\kappa_e \rho} = 0.02\ cm. \;:\; \text{mean free path}$$

Thomson electron scattering is the most common opacity in stellar interior where ~~ions~~ matters are fully ionized

It is a constant, independent of ρ, T,
In power-law notation

$$\kappa_e = \kappa_0 \rho^n T^{-s}$$

$$\kappa_0 = 0.2(1+X)$$

$$n = 0$$

$$s = 0$$

Exp: In Interstellar medium. $n_e = 10\ cm^{-3}$

$$\lambda_{ph} = \frac{1}{\sigma_e n_e} = \frac{1}{0.66 \times 10^{-24} \cdot 10} = 10^{23}\ cm$$

1 light year (Ly) $= 9.46 \times 10^{17}$ cm

$\lambda_{ph} = 10^5$ Ly

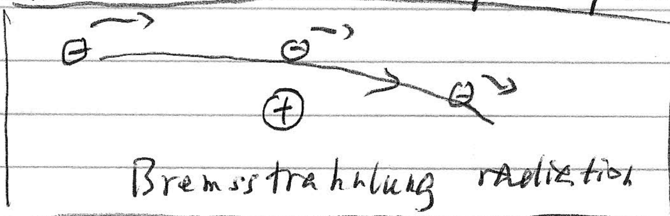
★ Free - Free Absorption κ_{ff}

Inverse process of free - free emission

Also called Bremsstrahlung radiation

produced by electron radiation due to Coulomb force of ion

electron accelerating



$$P(t) = \frac{2}{3}\frac{e^2}{c^3}a(t)$$: the Larmor result

$$4\pi j \rho = \frac{2\pi}{3}\frac{Z_c^2 e^6}{m_e c^3 \hbar}\left(\frac{2\pi k T}{m_e}\right)^{\frac{1}{2}} n_e n_I$$

Z_c : ion charge

$$\kappa_{ff} = 10^{23}\frac{\rho}{\mu_e}\cdot\frac{Z_c^2}{\mu_I} T^{-3.5} \quad cm^2 g^{-1} \text{ --- } (4.61)$$

$$\kappa_{ff} \sim \rho T^{-3.5}$$

$$\kappa_{ff} = \underbrace{\kappa_0 \rho T^{-3.5}}_{\text{so-called}} \quad \text{with} \quad \boxed{n=1, \ s=3.5}$$

so-called Kramer's opacity

High in stellar interior, fully ionized,

but prefer lower temperature

★ Bound - free Absorption :

In low - temperature area close to the surface ($T \sim 10^4 K$)

where neutral atoms start to form; partial ionization

$$\kappa_{bf} = 4 \times 10^{25} Z (1+X) \rho T^{-3.5} \quad cm^2 g^{-1} \quad (4.63)$$

$$n = 1, \quad s = 3.5$$

This absorption is to ionize the electron from an atom or ion by absorption of a photon

* Bound – Bound Absorption.
  photon-induced transition between bound levels in atoms
  $\Rightarrow$ forming spectral lines, but not continuum

* H^- opacity (Hydrogen-minus, or metal hydrogen)
  when temperature is very low, e.g., $T = 2000\,K$
  H, He are neutral : $\kappa_e = 0$, $\kappa_{ff} = 0$, $\kappa_{bf} = 0$
  Only metals are ionized, providing small amount of e^-.

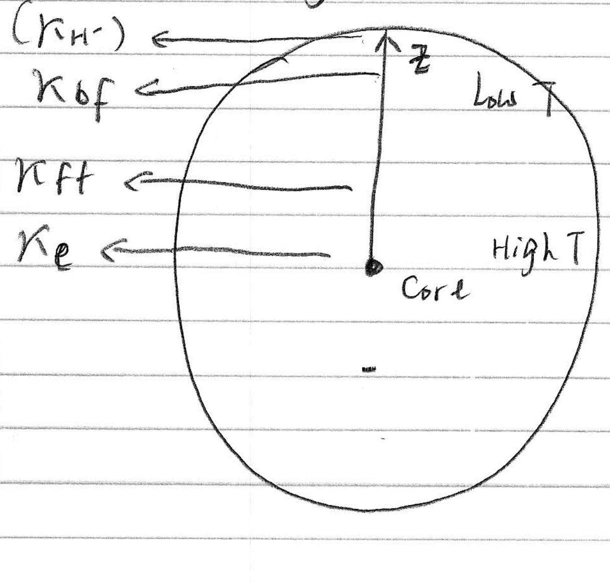
$$H + \underset{\text{from metals}}{e^-} = H^-$$

Ionization of H^- is only 0.75 eV
(Ionization of H to H^+) is 13.6 eV).

$$\kappa_{H^-} = 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{\frac{1}{2}} T^{9} \ cm^2 g^{-1} \quad \cdots 4.45$$

$$\boxed{n = \tfrac{1}{2}, \quad s = -9}$$

* Summary



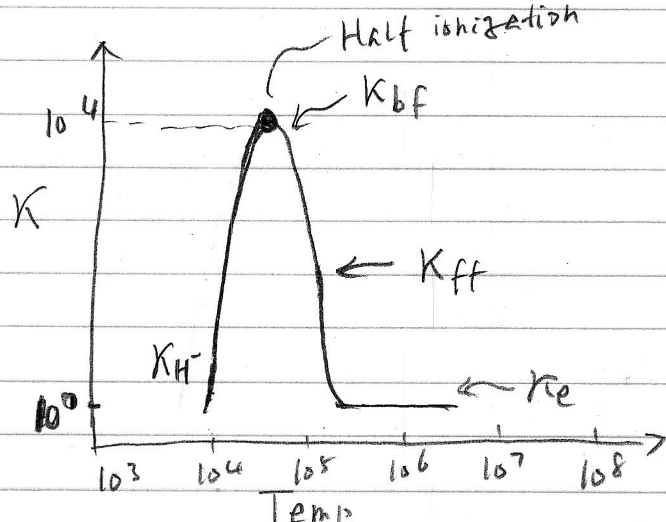


Fig 4.2 (P217)

CH5. Heat Transfer by Convection (motion of medium)

Energy $\quad V \neq 0$

Three ways of Energy Transfer

	①	②	③
	Conduction	Radiation	Convection
plasma	$V=0$	$V=0$	$V \neq 0$

All related with temperature gradient : $\propto \frac{dT}{dr}$

Energy goes from area of high T $\rightarrow$ low T

$$F = -D \frac{dT}{dr}$$: Fick's Law of Diffusion

F: Energy flux

Conduction: $D_{cond} = C_v V_e \lambda_e / 3 \xrightarrow{plasma} 9.2 \times 10^{-7} T^{\frac{5}{2}}$ erg $K^{-1} cm^{-1} s^{-1}$

Radiation $\quad D_{rad} = C_{ph} \, C \, \lambda_{ph} / 3 = \frac{4ac}{3} \frac{1}{k\rho} T^3$

$C_{ph} = 4aT^3$

Convection $\quad D_{rad} = C_p \rho W \quad \text{------} \quad (5\text{-}36)$

C_p: specific heat of constant pressure

W: average convection velocity

Condition to dominate:

①	②	③
Conduction	Radiation	Convection
High ρ	High T	High $\frac{dT}{dr}$

In main sequence stars, F_{rad} dominates

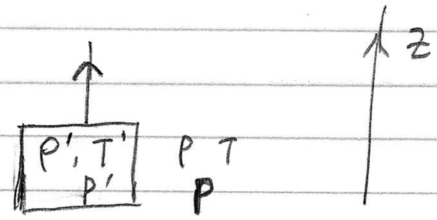
$$F_{rad} = -\frac{4ac}{3k\rho} T^3 \frac{dT}{dr}$$

If $\frac{dT}{dr} > \left(\frac{dT}{dr}\right)_{critical}$, higher than a critical temperature gradient, instability happens $\longrightarrow$ turbulent convection

Criteria for Convection (CH 5.1.1)

Convection is driven by $\boxed{\text{buoyancy force}}$

a common force in a fluid related with pressure & gravity



parcel or bubble

Consider a temperature disturbance in the parcel : $T' > T$

$$\boxed{P' = P} \quad \text{pressure always in equilibrium}$$

$$P = \frac{\rho}{\mu} N_A k T$$

$$\Rightarrow \rho' < \rho \quad \text{density smaller}.$$

$$F = (\rho V g - \rho' V g) = (\rho - \rho') V g : \text{~~net~~ buoyancy force}$$

$$F = \frac{dP'}{dr} - \rho' g = \frac{dP}{dr} - \rho' g = \rho g - \rho' g > 0$$

The parcel rises up.

Whether the parcel continues the rising or not,

depending on $\frac{dT}{dr}$; At higher positions

$T' > T$: always T' larger → convection

$T' < T$: $\Rightarrow \rho' > \rho$ → parcel sinks

→ no convection

Thus, the criterion is

$$\boxed{\frac{dT'}{dz} > \frac{dT}{dz}}, \text{ or } \boxed{\left|\frac{dT'}{dz}\right| < \left|\frac{dT}{dz}\right|} \Rightarrow T' > T$$

$\frac{dT'}{dz} < 0$, $\frac{dT}{dz} < 0$ ---- (5.8)

(Continued)

$$\frac{dT}{dz} = -F \cdot \frac{3K\rho}{4ac} \frac{1}{T^3}$$ From radiation model

determined by F (or Luminosity), $\underline{K}$, ρ, T

$\left(\frac{dT'}{dz}\right)$ is calculated from the adiabatic process, assuming no heat exchange (ideal case)

$$\frac{dT'}{dz} = \left(\frac{dT'}{dz}\right)_{ad}$$

* For the convenience, introduce

$$\beta = -\frac{dT}{dz}$$: T gradient parameter ; $\beta > 0$

$$\nabla = \left(\frac{\partial \ln T}{\partial \ln P}\right) = \frac{\lambda_P}{\lambda_T}$$: dimensionless temperature gradient ; $\nabla > 0$

$$T = T_0 e^{-\frac{z}{\lambda_T}} \quad d\ln T = \frac{dT}{T} = \frac{T_0 e^{-\frac{z}{\lambda_T}} \left(-\frac{1}{\lambda_T}\right) dz}{T} = -\frac{dz}{\lambda_T}$$

* Criteria: $\beta > \beta_{ad}$ ------ (5.9)

$\nabla > \nabla_{ad}$ (5.10)

* Relate β and ∇

$$\frac{dT}{dz} = -\beta = \frac{dT}{dP}\frac{dP}{dz} = \frac{d\ln T}{d\ln P} \cdot \frac{T}{P} \cdot \frac{dP}{dz} = \nabla \cdot \frac{T\, d\ln P}{dz} = -\nabla \frac{T}{\lambda_P}$$

$$\frac{d\ln P}{dz} = -\frac{1}{\lambda_P}$$

$$\Rightarrow \beta = -\frac{dT}{dz} = \frac{T}{\lambda_P}\nabla$$

* ∇_{ad}: $\nabla_{ad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_{ad} = \frac{\Gamma_2 - 1}{\Gamma_2}$ ------ (3.94)

Ideal gas : $\Gamma_2 = \frac{5}{3}$

$$\boxed{\nabla_{ad} = \frac{\frac{5}{3}-1}{5/3} = \frac{2}{5} = 0.4}$$

Mixture gas. $\nabla_{ad} = \frac{2(4-3\beta)}{32 - 24\beta - 3\beta^2}$, where $\boxed{\beta = \frac{P_g}{P_{total}}}$ (gas β)