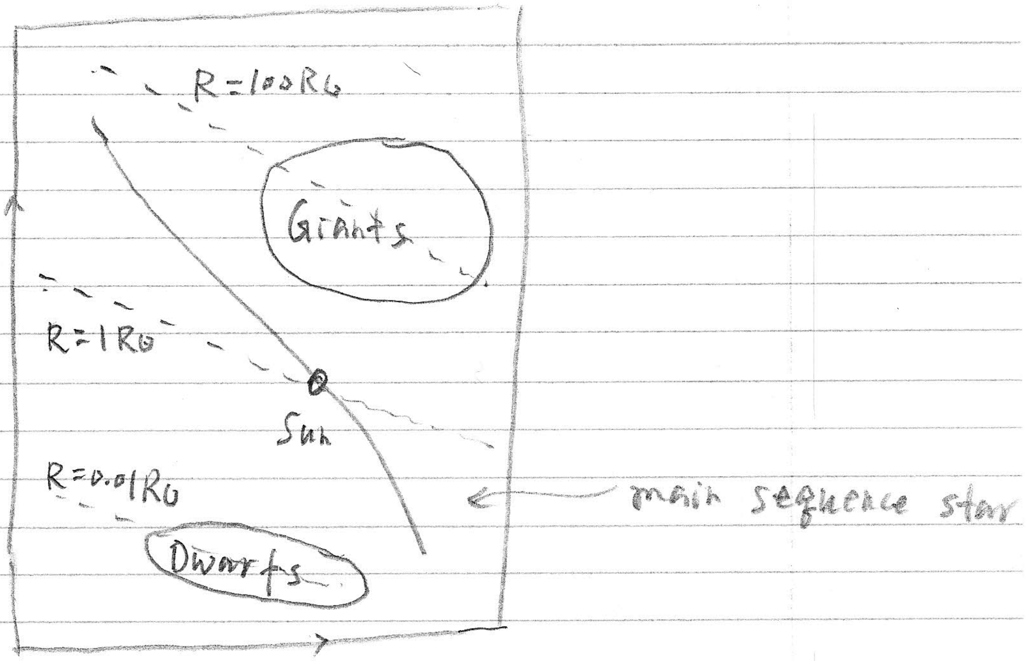
Feb. 9, 2011 ①

* stellar Evolution Overview - the basics (part-1) (Appendix-A)
* Governing Equations of stellar structure & Evolution (CH7.1)
* Hertzsprung - Russell (HR) diagram

① apparent magnitude (M_V)

② Absolute magnitude (M_V)

③ Luminosity (L)



① color index (observation)
small — large

② spectral class (observation)
O - B - A - F - G - K - M

③ Effective temperature (T_{eff}) (theory)
~30,000 K — 3,000 K

$$L = 4\pi R^2 \sigma T_{eff}^4$$

σ : stefan - Boltzmann constant $= 5.67 \times 10^{-8}\ W\,m^{-2}\,K^{-4}$ (MKS unit)
$= 5.67 \times 10^{-5}\ erg\ cm^{-2}\,K^{-4}\,s^{-1}$ (cgs) unit

In log scale

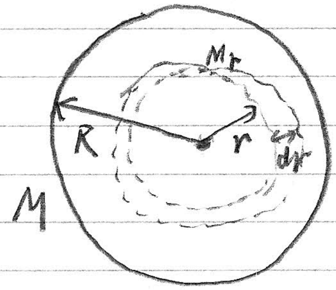
$$\log L = \log(4\pi R^2) + \log(\sigma T_{eff}^4)$$

Different R → straight lines in HR diagram

★ Governing Equations of stellar structure
Top-down Approach

★ Assumption:

A spherically symetric, non-rotating, non-magnetic, single object



→ Independent variable: r, $\in [0, R]$ (center, Radius)
or conveniently $M_r \in [0, M]$ (total mass)

M_r: mass within r

★ Governing Equations from universal laws

1. conservation of mass
2. conservation of momentum
3. conservation of energy

in differential format

1. Conservation of mass

general format in fluid dynamics

$$\frac{d\rho}{dt} + \rho(\nabla \cdot \vec{V}) = 0$$

ρ: density
$\vec{V}$: velocity

$$\frac{\partial \rho}{\partial t} + (\vec{V} \cdot \nabla)\rho + \rho(\nabla \cdot \vec{V}) = 0 \Rightarrow \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

$\frac{d}{dt}$: total
$\frac{\partial}{\partial t}$: partial derivative
$\frac{d}{dt} = \frac{\partial}{\partial t} + (\vec{V} \cdot \nabla)$

specific in stars

$$dM_r = \rho(r) 4\pi r^2 dr$$

$$dr = \frac{1}{\rho(r) 4\pi r^2} \cdot dM_r \Rightarrow dr \text{ and } dM_r \text{ inter-changable}$$

either r or M_r can be independent variable

$$M = \int_0^R 4\pi r^2 \rho(r) dr$$

(2) Conservation of Momentum

General $$\rho \frac{d\vec{v}}{dt} = -\nabla P + \rho \vec{g}$$

$$g(r) = \cancel{\frac{GM(r)}{r^2}} = \frac{GM_r}{r^2}$$: universal law of gravitation

Specific $$\nabla P = \rho \vec{g}$$

$$\frac{dP(r)}{dr} = -\rho(r) g(r) = -\rho(r) \frac{GM_r}{r^2} \quad \text{---} \quad (1.6)$$

or $$\frac{dP(r)}{dM_r} = -\frac{GM_r}{4\pi r^4} \quad \text{------} \quad (7.5)$$

(3) Conservation of energy

general $$\rho \frac{dE}{dt} + P \nabla \cdot \vec{v} = \nabla \cdot (\vec{K} \cdot \nabla T) + Q_{source} - Q_{sink}$$

E: internal energy. $\vec{K}$: conductivity tensor

Q_{source}: energy source, e.g. nuclear fusion
Joule heating

Q_{sink}: energy sink: radiation

Specific: $$\frac{dL(r)}{dr} = 4\pi r^2 \rho(r) \varepsilon(r) \quad \text{---} \quad (7.7)$$

$L(r)$: Luminosity, $\varepsilon(r)$: energy generation rate (nuclear) or $$\frac{dL(r)}{dM_r} = \varepsilon \qquad \text{---} \quad (7.7)$$

If $\varepsilon = 0$, no nuclear energy, e.g. in radiative layer

$$\frac{dL(r)}{dr} = 0.$$

$$\Rightarrow \nabla = \frac{d\ln T}{d\ln P} = \nabla_{rad} \quad \text{if } \nabla_{rad} \leq \nabla_{ad}$$

∇: local slope of temperature with respect to pressure

∇_{rad}: temperature slope due to radiation diffusion

$$\nabla_{rad} = \frac{3}{16\pi a c}\frac{\rho \kappa}{T^4}\frac{L(r)}{G M_r} \qquad \text{---} \quad (7.8)$$

— details in Chap. 4.

$$\Rightarrow \nabla = \nabla_{ad} \quad \text{if } \nabla_{rad} > \nabla_{ad}$$

∇_{ad}: adiabatic temperature gradient

$\Rightarrow$ convection (details in Chap. 5)

Unknown variables: $T(r)$: Four variables
$\rho(r)$ Three equations
$P(r)$
$L(r)$

Constituent equations : determined by microphysics

① Equation of state

$$P = P(\rho, T)$$

In stars. $P = P(\rho, T, X)$ (details Chap 3)

X : composition of hydrogen, helium and metals

\# ~~Free~~ Controlling parameters ; determined from microphysics

$\varepsilon = \varepsilon(\rho, T, X)$

ε : energy generate due to thermal nuclear fusion
(details in chap. 6)

$\kappa = \kappa(\rho, T, X)$

κ: Radiative opacity (details in chap. 4)

\# Three governing differential equations
+ One constituent equation + using ε, κ
$\Rightarrow$ solve numerically for the four variables
$L(r)$, $P(r)$, $\rho(r)$, $T(r)$
or $L(M_r)$, $P(M_r)$, $\rho(M_r)$, $T(M_r)$.

(details in Chap. 7)

boundary condition : At $r=0$, $M_r = 0$, $L_r = 0$

At $r = R$, $M_r = M$, $\rho = T = 0$, $L_r = L$

Analytic method : polytropic equations

Numerical method : hands-on experience $\rightarrow$ project