Lect. 25; May 4, 2010 ①

Reflection and Refraction of EM waves (CH 24)

Explain: phase change of reflection: π

Snell's Law of refraction, $\sin\theta_c = \frac{n_1}{n_2}$

Stoke's relations on r and t

Brewster's Law on Polarization: $\theta_p = \tan^{-1}\frac{\theta_2}{\theta_1}$

Consider the plane wave at the interface of two dielectr[ics]

Mx Eqs:

$$\begin{cases} ① & \nabla\cdot\vec{D} = 0 \\ ② & \nabla\cdot\vec{B} = 0 \\ ③ & \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ ④ & \nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t} \end{cases} \Rightarrow \begin{cases} ① & D_{1n} = D_{2n} \\ ③ & B_{1n} = B_{2n} \\ ③ & E_{1t} = E_{2t} \\ ④ & H_{1t} = H_{2t} \end{cases}$$

and

$$\vec{D}_1 = \varepsilon_1\vec{E}_1$$
$$\vec{D}_2 = \varepsilon_2\vec{E}_2$$
$$\vec{H}_1 = \vec{B}_1/\mu_1$$
$$\vec{H}_2 = \vec{B}_2/\mu_2$$

two media: (ε_1, μ_1)
(ε_2, μ_2)

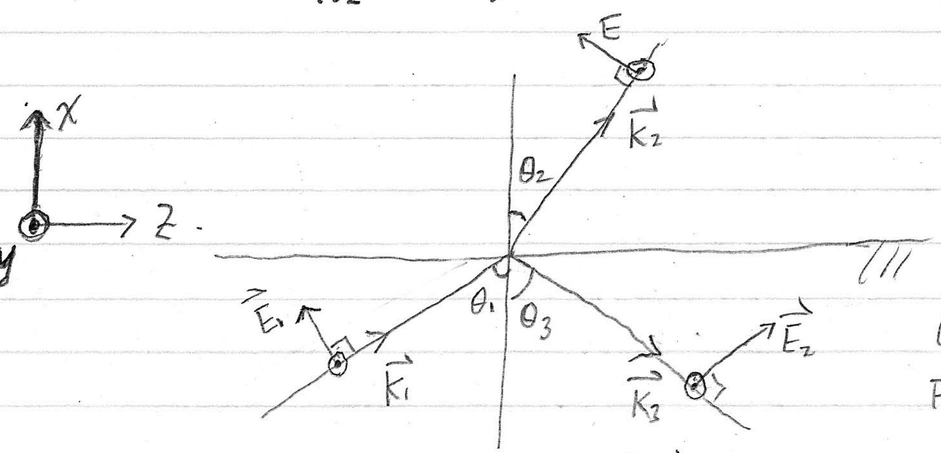


Fig. 24.1

plane wave:

$$\begin{cases} \vec{E}_1 = \vec{E}_{10}\, e^{i(\vec{k}_1\cdot\vec{r} - \omega t)} \\ \vec{E}_2 = \vec{E}_{20}\, e^{i(\vec{k}_2\cdot\vec{r} - \omega t)} \\ \vec{E}_3 = \vec{E}_{30}\, e^{i(\vec{k}_3\cdot\vec{r} - \omega t)} \end{cases} \quad \text{--- (1)}$$

EM waves at an interface of two dielectrics (CH 24.2) (continued)

From the four MX Eqs in a homogeneous medium ⇒

$$\begin{cases} k_1^2 = \omega_1^2 \varepsilon_1 \mu_1 \\ k_2^2 = \omega_2^2 \varepsilon_2 \mu_2 \\ k_3^2 = \omega_3^2 \varepsilon_1 \mu_1 \end{cases} \quad \text{-------- (2)}$$

From (CH 23-2)

Assuming $k_y = 0$, $\hat{k}$ in (x, z) plane, the incidence plane,
This does not loss the generality of discussion

$$k_{1y} = k_{2y} = k_{3y} = 0$$

$$\vec{k}_1 \cdot \vec{r} = k_{1x} x + k_{1z} z$$

At the interface: $\vec{k}_1 \cdot \vec{r} = k_{1z} z$ =
$x = 0$ $\quad \vec{k}_2 \cdot \vec{r} = k_{2z} z$
$\vec{k}_3 \cdot \vec{r} = k_{3z} z$

Sine $\vec{E}$ tangential component continuous, "$E_{1t} = E_{2t}$"

$$E_{1z} e^{i(k_{1z} z - \omega_1 t)} + E_{3z} e^{i(k_{3z} z - \omega_3 t)} = E_{2z} e^{i(k_{2z} z - \omega_2 t)}$$

This has to satisfy for any (z, t).
Therefore, the <u>phase term</u> has to be the same

$$k_{1z} z - \omega_1 t = k_{3z} z - \omega_3 t = k_{2z} z - \omega_2 t$$

For any t: $\omega_1 = \omega_2 = \omega_3 = \omega$ ---- (3)

For any z: $k_{1z} = k_{2z} = k_{3z}$ ---- (4)

or $k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3$ ---- (4)

Since $\omega_1 = \omega_3 \Rightarrow k_1 = k_3 \Rightarrow \theta_1 = \theta_3$

Therefore, the incidence angle equals the reflection angle

EM wave at the interface (CH 24.2) (continued)

$$k_1 \sin\theta_1 = k_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \frac{\omega_2\sqrt{\varepsilon_2\mu_2}}{\omega_1\sqrt{\varepsilon_1\mu_1}} = \frac{\frac{1}{v_2}}{\frac{1}{v_1}} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \quad \cdots (11)$$

where $n_1 = \frac{c}{v_1} = c\sqrt{\varepsilon_1\mu_1}$

Therefore, Snell's Law is required by the phase continuity.

Now, consider the amplitude : $\begin{cases} \text{Case 1: } E_{\parallel} \text{ parallel} \\ \text{Case 2: } E_{\perp} \text{ perpendicular} \end{cases}$

with respect to the plane of incidence

Case 1: E parallel to the plane of incidence

$\vec{E}$ tangential continuous, the amplitude term

$$E_{1z} + E_{3z} = E_{2z}$$

$$-E_{10}\cos\theta_1 + E_{30}\cos\theta_1 = -E_{20}\cos\theta_2 \quad \cdots\cdots (13)$$

$\vec{D}$ normal component continuous "$D_{1n} = D_{2n}$"

$$D_{1x} + D_{3x} = D_{2x}$$

$$\varepsilon_1 E_{1x} + \varepsilon_3 E_{3x} = \varepsilon_2 E_{2x}$$

$$\varepsilon_1 E_{10}\sin\theta_1 + \varepsilon_1 E_{30}\sin\theta_3 = \varepsilon_2 E_{20}\sin\theta_2 \quad \cdots (16)$$

Combine (13) + (16), solve for E_{20} and E_{30}

From (13), $E_{20} = \frac{(E_{10} - E_{30})\cos\theta_1}{\cos\theta_2}$

Substitute E_{20} in (16)

$$\varepsilon_1 (E_{10} + E_{30})\sin\theta_1 = \varepsilon_2 \sin\theta_2 \frac{E_{10} - E_{30}}{\cos\theta_2} \cdot \cos\theta_1$$

EM wave at the interface (CH 24-2) continued)

Amplitude Reflection Coefficient for parallel $E_{\parallel}$: $r_{\parallel}$

$$r_{\parallel} = \frac{E_{30}}{E_{10}} = \frac{\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \quad \text{---- (17)}$$

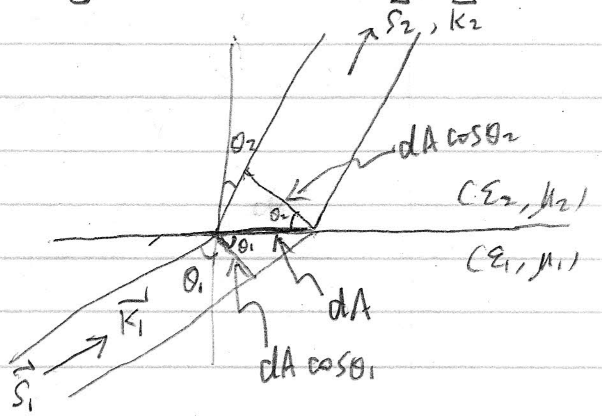
Amplitude Transmission Coefficient for parallel $E_{\parallel}$: $t_{\parallel}$

$$t_{\parallel} \approx \frac{E_{20}}{E_{10}} = \frac{2\varepsilon_1 \sin\theta_1 \cos\theta_1}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \quad \text{---- (18)}$$

"Intensity" Reflection coeffient $R_{\parallel}$ for $E_{\parallel}$

Transmission Coefficient $T_{\parallel}$

Have to consider the poynting vector $\vec{S} = \vec{E} \times \vec{H}$ and the projection effect



Incident energy: $S_1 \cdot dA \cdot \cos\theta_1$

Reflection energy: $S_3\, dA \cos\theta_1$

Refraction energy: $S_2\, dA \cos\theta_2$

$$R_{\parallel} = \frac{S_3\, \cancel{dA \cos\theta_1}}{S_1\, \cancel{dA \cos\theta_1}} = \frac{S_3}{S_1} = \frac{\langle \vec{E_3} \times \vec{H_3} \rangle}{\langle \vec{E_1} \times \vec{H_1} \rangle} = \frac{\sqrt{\varepsilon_1/\mu_1}\, |E_{30}|^2}{\sqrt{\varepsilon_1/\mu_1}\, |E_{10}|^2}$$

$$R_{\parallel} = \left| \frac{E_{30}}{E_{10}} \right|^2 = r_{\parallel}^2 \quad \text{--- (20)}$$

$$T_{\parallel} = \frac{S_2 \cdot dA \cos\theta_2}{S_1 \cdot dA \cos\theta_1} = \frac{\langle \vec{E_2} \times \vec{H_2} \rangle \cos\theta_2}{\langle \vec{E_1} \times \vec{H_1} \rangle \cos\theta_1} = \frac{\sqrt{\varepsilon_2/\mu_2}\, |E_{20}|^2 \cos\theta_2}{\sqrt{\varepsilon_1/\mu_1}\, |E_{10}|^2 \cos\theta_1}$$

$$T_{\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \cdot \frac{\cos\theta_2}{\cos\theta_1} \frac{|E_{20}|^2}{|E_{10}|^2} = \frac{\sin\theta_1 \cos\theta_2}{\sin\theta_2 \cos\theta_1} t_{\parallel}^2 \quad \text{--- (21)}$$

~~(Note: mistake in the book)~~

It can be shown: $R_{\parallel} + T_{\parallel} = 1$ --- (22)

EM wave at the interface (CH 24.2) (continued)

$r_{\parallel}$, $t_{\parallel}$ can be re-written as

$$r_{\parallel} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \qquad \text{---- (24)}$$

$$t_{\parallel} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)} \qquad \text{-- (25)}$$

(a) $n_2 = n_1$, no reflection

$\theta_2 = \theta_1$

$r_{\parallel} = 0$: no reflection

$t_{\parallel} = 1$: full transmission

(b) Brewster's law : polarization by reflection

$\theta_1 + \theta_2 = \frac{\pi}{2}$

$\tan(\theta_1 + \theta_2) = \infty$

$r_{\parallel} = 0$: no reflection of $E_{\parallel}$

Brewster's angle θ_p:

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin\theta_p}{\sin(\frac{\pi}{2}-\theta_p)} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

(c) phase change

incident on denser medium. $\theta_1 > \theta_2$, $\theta_1 - \theta_2 > 0$

If $\theta_1 + \theta_2 > \frac{\pi}{2}$, or $\theta_1 > \theta_p$, $\tan(\theta_1+\theta_2) < 0$

$r_{\parallel} < 0$.

The direction of $\vec{E}$ flipped, resulting in π-phase change

For $E_{\perp}$, if $n_2 > n_1$, $r_{\perp} < 0$ always.

\# EM wave at the interface (CH 24.2) (continued)

Normal Incidence: $\theta_1 = \theta_2 = 0$

$$r_{\parallel} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_1} = \frac{n_2 - n_1}{n_2 + n_1}$$

$$R_{\parallel} = r_{\parallel}^2 = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2$$

\# Case 2: E perpendicular to the plane of incidence: $E_{\perp}$

E_t continuous $\quad E_{10} + E_{30} = E_{20}$ ------ (58)

H_t continuous: $H_{10}\cos\theta_1 - H_{30}\cos\theta_1 = H_{20}\cos\theta_2$ --- (60)

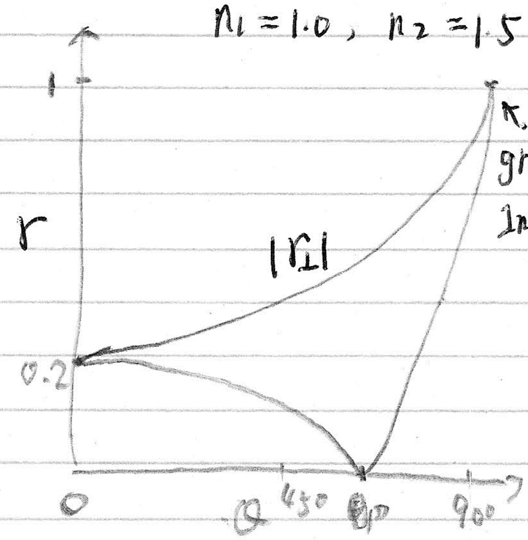
$$\Rightarrow \begin{cases} r_{\perp} = -\dfrac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \\[2ex] t_{\perp} = \dfrac{2\sin\theta_2 \cos\theta_1}{\sin(\theta_1 + \theta_2)} \end{cases}$$

If $\theta_1 < \theta_2$, or $n_1 < n_2$, $r_{\perp} < 0$, π-phase change

\# Summarize: Fig. 24.8, Fig 24.9

Amplitude

$n_1 = 1.0$, $n_2 = 1.5$



Phase