

Lect. 24, April 29, 2010

①

Plane EM wave properties (CH 23.2)

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{H} = \vec{H}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

\vec{E}_0, \vec{H}_0 are independent of (\vec{r}, t) ; space-time

$$\vec{k} \cdot \vec{r} = k_x x + k_y y + k_z z$$

Eq. 1: $\nabla \cdot \vec{E} = 0 \Rightarrow i\vec{k} \cdot \vec{E} = 0$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = ik_x E_x + ik_y E_y + ik_z E_z = i(\vec{k} \cdot \vec{E})$$

where $E_x = E_{0x} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$

Therefore $\vec{k} \cdot \vec{E} = 0$ (18)

Eq. 2 Similarly: $\nabla \cdot \vec{H} = 0 \Rightarrow \vec{k} \cdot \vec{H} = 0$ (19)

Eq. 3 $\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$

LHS: $\nabla \times \vec{E} = i\vec{k} \times \vec{E}$

RHS: $-\mu_0 \frac{\partial \vec{H}}{\partial t} = i\omega \mu_0 \vec{H}$, $\frac{\partial \vec{H}}{\partial t} = -i\omega \vec{H}$

$$\Rightarrow \vec{H} = \frac{\vec{k} \times \vec{E}}{\omega \mu_0} \quad (20)$$

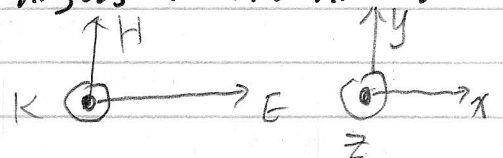
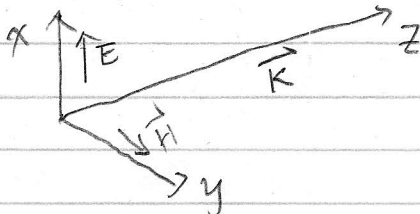
Eq. 4. Similarly $\nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$

~~$i\omega \mu_0 \vec{H}$~~

$$i\vec{k} \times \vec{H} = \epsilon_0 (-i\omega) \vec{E}$$

$$\vec{E} = \frac{\vec{H} \times \vec{k}}{\omega \epsilon_0} \quad (21)$$

Therefore, $\vec{E}, \vec{H}, \vec{k}$ are at right angles to one another



PLANE EM WAVE properties (CH 23.2). (Continued)

$$\text{Eq. 21} \Rightarrow H_0 = \frac{k}{\omega \mu_0} E_0$$

$$\text{Eq. 22} \Rightarrow E_0 = \frac{k}{\omega \epsilon} H_0$$

$$\text{Combine the two: } \left(\frac{k}{\omega}\right)^2 = \epsilon \mu_0$$

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon \mu_0}} \quad \text{again, the wave velocity - (27)}$$

Note: since k, ϵ, μ_0 are real numbers.

$$\vec{E} \text{ and } \vec{H} \text{ are in phase; } \vec{E} = 0 \Leftrightarrow \vec{H} = 0$$

In free space, $\epsilon_0 \neq 0, \epsilon_0 \neq \infty$
 $\mu_0 \neq 0, \mu_0 \neq \infty$

The speed of EM wave is finite, not infinite

