Lect. 22 ; Apr. 22, 2010

Stokes' Parameters (Supplement).

Quantity the state of polarization of the light

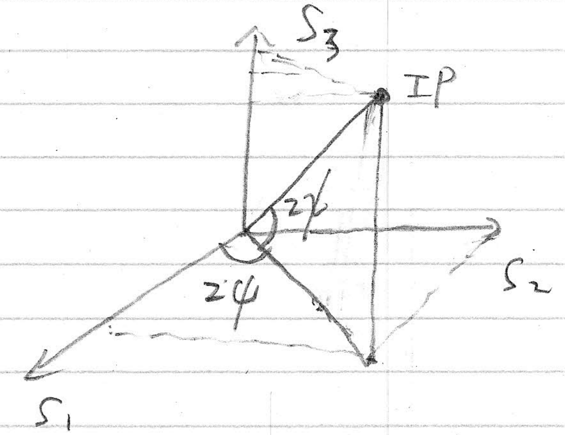
$$S = (S_0, S_1, S_2, S_3) = (I, Q, U, V)$$

$$\begin{cases} S_0 = I & : \text{total Intensity} \\ Q = S_1 = \underline{Ip} \cdot \cos 2\psi \cos 2\chi & : \text{linear polarization} \\ U = S_2 = Ip \cdot \sin 2\psi \cos 2\chi & : \text{linear polarization at } 90^\circ \\ V = S_3 = Ip \cdot \sin 2\chi & : \text{circular polarization} \end{cases}$$

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{I} \quad : \text{degree of polarization}$$

$$\tan 2\chi = \frac{S_3}{\sqrt{S_1^2 + S_2^2}}$$

$$\tan 2\psi = \frac{S_2}{S_1}$$



Poincaré sphere

$\sqrt{Q^2 + U^2 + V^2}$: total polarization

Double Refraction (CH 22.5)

Also called "Birefringence", is the decomposition of a ray of light into two rays when it passes through certain type of materials, e.g., calcite crystals ($CaCO_3$)

Two rays:

1. Ordinary ray: n_o, $V_o = \frac{c}{n_o}$
2. Extraordinary ray: n_e, $V_e = \frac{c}{n_e}$

Calcite: at 4046 Å, $n_o = 1.681$; $n_e = 1.497$

* It is caused by the structure of the crystal (anisotropic) (CH 22.12)

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 \vec{E} + \vec{P}$$ $\vec{E}$: electric field

$\vec{D}$: Electric Displacement — ε: Dielectric permitivity

For isotropic material ε = constant scalar

For anisotropic material ε = tensor $\overleftrightarrow{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$

$$\vec{D} = \overleftrightarrow{\varepsilon} \cdot \vec{E}$$

For the three principle axises, ε_x, ε_y, ε_z

* Uniaxial Anisotropic medium

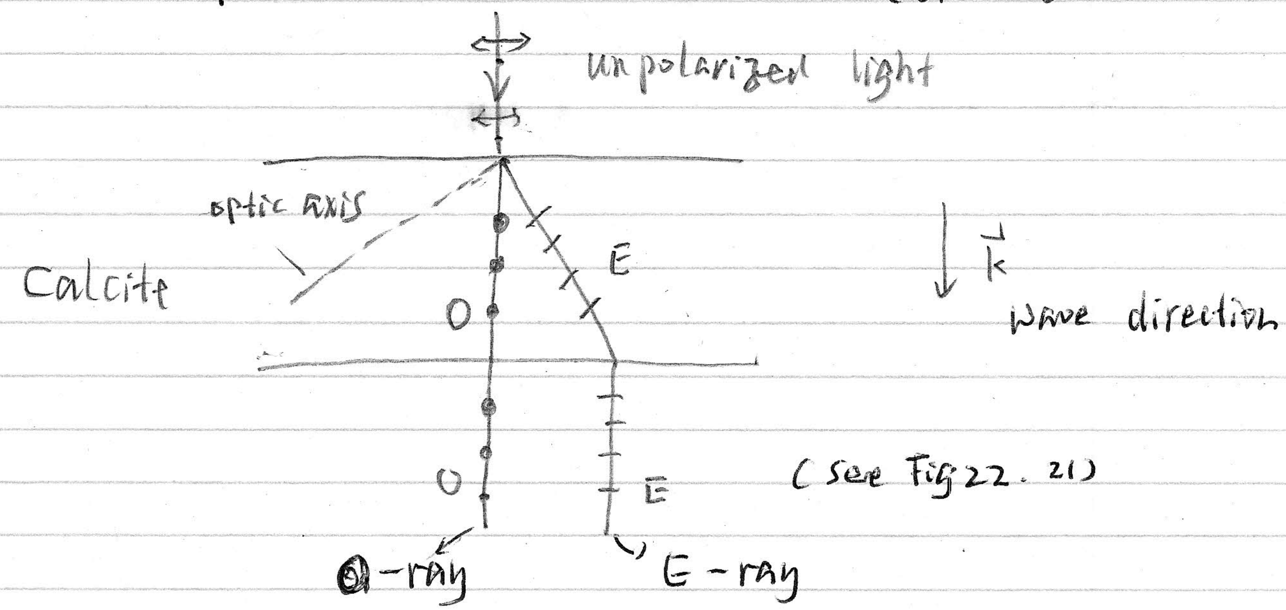
$$\varepsilon_x = \varepsilon_y \neq \varepsilon_z$$

then Z: represents the <u>optic axis</u> of the medium

$$n_o = \sqrt{\frac{\varepsilon_x}{\varepsilon_0}} = \sqrt{\frac{\varepsilon_y}{\varepsilon_0}}$$

$$n_e = n_z = \sqrt{\frac{\varepsilon_z}{\varepsilon_0}}$$

Double Refraction - normal incidence (CH 22.5.1)



O-ray : $\vec{E}$ vector $\perp$ to ($\vec{K}$ and optic axis)

E-ray : $\vec{E}$ vector on the plane of ($\vec{K}$ and optic axis)

E-ray is ~~faster~~ than the O-ray.

Along the optic axis, O-ray and E-ray have same speed

Perpendicular to the optic axis $\begin{cases} V_o = c/n_o \\ V_e = c/n_e \end{cases}$

For calcite, $V_e > V_o$

For any angle θ. between $\vec{K}$ and optic axis

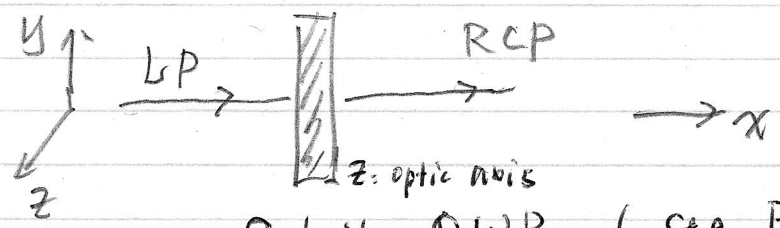
$$V_o < V_{re} < V_e$$

$$\frac{1}{V_{re}^2} = \frac{\sin^2\theta}{(c/n_e)^2} + \frac{\cos^2\theta}{(c/n_o)^2} \quad \text{------} \quad (37)$$

Therefore, two linearly polarized lights emerged from calcite crystal due to double refraction

六 Interference of Polarized Light (CH 22.6)

- Quarter Wave plate (QWP)
- Half wave plate (HWP)



Calcite QWP (see Fig 22.25)

Linear polarized wave to circularly polarized wave

- Incident light: Linearly polarized

$E_y = E_0 \sin\phi \cos(kx - \omega t)$: O-ray : $\perp$ optic axis

$E_z = E_0 \cos\phi \cos(kx - \omega t)$: E-ray : $\parallel$ optic axis

ϕ: angle between optic axis and linear Pol. direction

- In the Calcite, $k \rightarrow nk$, since $\lambda = \frac{\lambda_0}{n}$

$E_y = E_0 \sin\phi \cos(\underbrace{n_o kx - \omega t}_{})$: $\theta_o - \omega t$

$E_z = E_0 \cos\phi \cos(\underbrace{n_e kx - \omega t}_{})$: $\theta_e - \omega t$

phase difference $\theta = \theta_o - \theta_e = n_o kx - n_e kx$

$$\theta = \frac{\omega}{c}(n_o - n_e)d$$

d: thickness of the crystal

$\theta = \frac{\pi}{2} \Rightarrow$ QWP: Quarter Wave plate

$\theta = \pi \Rightarrow$ HWP: half wave plate

Electromagnetic Waves ; EM waves (CH23)

Described by Maxwell's equations

A set of four equations (in differential term)

(1) $\nabla \cdot \vec{D} = \rho$ ------- (1)

D : electric displacement

ρ : density of free electric charges

(2) $\nabla \cdot \vec{B} = 0$ ---- (2)

B : magnetic induction

(3) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ------- (3)

$\vec{E}$: electric field ; $V = E \cdot d$ (voltage)

(4) $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$ --- (4)

$\vec{J}$: electric current $\vec{J} = n_q \vec{U}_q$

$\vec{H}$: magnetic field

where do these equation originate? (CH 23.9)

Eq. (1) + (2) from "Gauss" Law

Eq. (3) from "Faraday"'s Law

Eq. (4) from "Ampere"'s Law

Eq (1) : Gauss's Law (1835):

" The ~~electric~~ electric flux through any closed surface is proportional to the enclosed electric charge "

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

Therefore, $\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \int_V \rho \, dV$

using the Divergence Theorem: $\oint \vec{E} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{E}) \, dv$

$\Rightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$, since $\vec{D} = \varepsilon_0 \vec{E}$ in free space

$\Rightarrow \nabla \cdot \vec{D} = \rho$