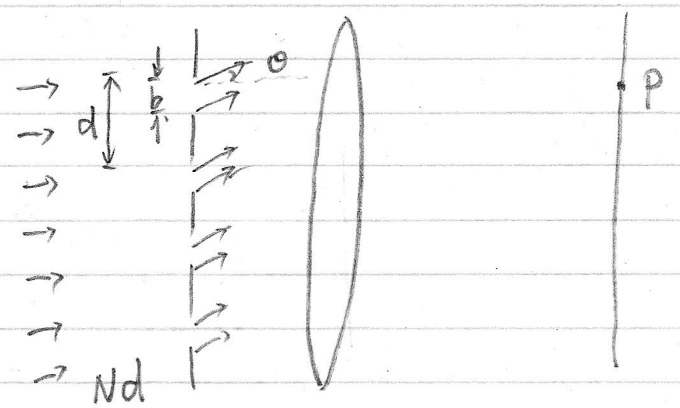
lect. 20,

N-slit Fraunhofer Diffraction Pattern (CH 18-7)



$E = E_1 + E_2 + \cdots E_N$: N-slit

$E = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$ first slit

$+ A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$ 2nd slit

$+ A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - (N-1)\Phi_1)$ ... Nth slit

$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta$

Use complex representation and partial sum of series (in single-slit)

$$E = \frac{A\sin\beta}{\beta}\cdot\frac{\sin N\gamma}{\sin\gamma}\cos\left[\omega t - \beta - \tfrac{1}{2}(N-1)\Phi_1\right]$$

$$\beta = \frac{\pi b\sin\theta}{\lambda}$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d\sin\theta.$$

$$\Rightarrow I = I_0 \frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

$\frac{\sin^2\beta}{\beta^2}$: diffraction by a single slit

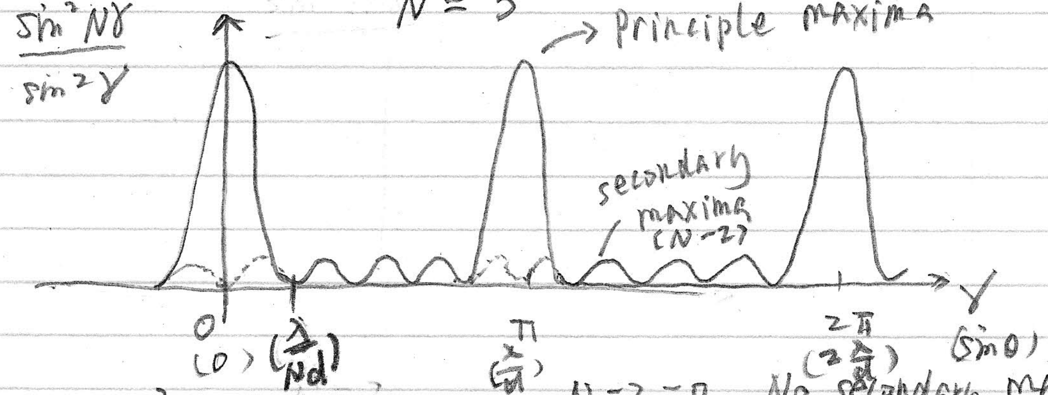
$\frac{\sin^2 N\gamma}{\sin^2\gamma}$: Interference pattern by N point sources

$N=1$, $I = I_0\frac{\sin^2\beta}{\beta^2}$ → single slit

$N=2$, $I = I_0\frac{\sin^2\beta}{\beta^2}(4\cos^2\gamma)$ → double-slit

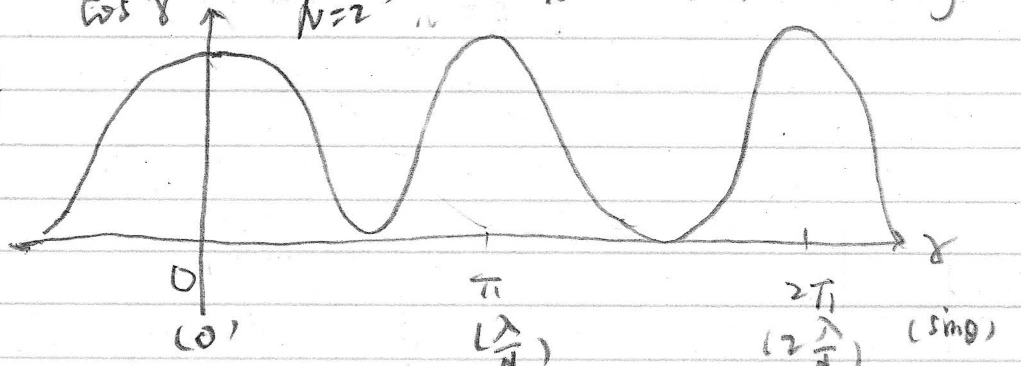
\# The fringe pattern – positions of maxima and minima (continued)

N-slit



$N-2=0$. No secondary maxima

Double slit



\# Principle maxima:

$$\sin\gamma = 0 \Rightarrow \gamma = m\pi$$

$$d\sin\theta = m\lambda, \quad m = 0, 1, 2 \cdots \qquad (51)$$

$$\lim_{\substack{\sin\gamma\to 0\\ \gamma\to m\pi}} \frac{\sin N\gamma}{\sin\gamma} = N$$

$$I = I_0 N^2 \frac{\sin^2\beta}{\beta}$$: all slits are in phase as $\gamma = m\pi$

\# Minima: Diffraction minima; $\beta = n\pi$; $b\sin\theta = n\lambda$, $n = 1, 2, \cdots$

Interference minima: $N\gamma = p\pi$. p integer, but $\neq N, 2N$

Between two principle maxima: $N-1$ interference minima

$N-2$ secondary maxima

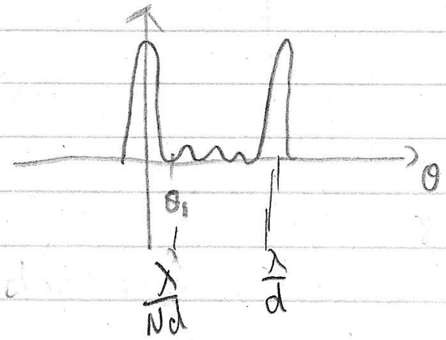
$$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \cdots \frac{(N-1)\lambda}{N}, \text{max}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N} \cdots$$

Width of the Principle Maxima (CH 18.7.2)

First minimum from the central principle maximum.

$$\cancel{d\theta} = d \sin\theta_1 = \frac{\lambda}{N}$$

$$\theta_1 \approx \sin\theta_1 = \frac{\lambda}{Nd}$$



Thus, the width is

$$\Delta\theta \approx \theta_1 = \frac{\lambda}{Nd}$$

N large, e.g., 10,000, $\Delta\theta$ is extremely small

For mth order principle maximum, the width

$$\Delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}$$

and $\quad d\sin\theta_m = m\lambda$

Diffraction Grating (CH 18.8)

An arrangement which consists of a large number of equidistant slits.

It produces grating spectrum. Grating equation:

$$\boxed{d \sin\theta_m = m\lambda} \quad \text{------} \quad (64)$$

θ_m: mth order principle maximum

θ_m depends on λ

$\Delta\theta_m$ is extremely small for large N at fixed λ.

Different λ corresponds to different diffraction angle θ_m.

Typical N: 15000 slits per inch

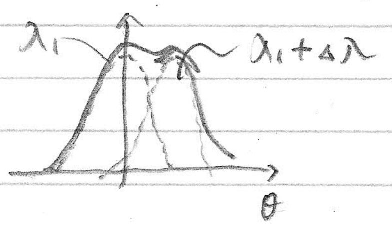
Dispersion

$$\frac{\Delta\theta}{\Delta\lambda} = \frac{m}{d\cos\theta},$$

$d \downarrow$, $\Delta\theta/\Delta\lambda \uparrow$

Since $\Delta\theta = \frac{\lambda}{Nd\cos\theta} \Rightarrow$ spectrum resolution $\Delta\lambda = \frac{\lambda}{Nm}$

Resolving Power of a Grating (18-8.2)



Rayleigh criterion: just resolved

the maximum of $\lambda_1 + \Delta\lambda$ falls on the first minimum of λ

First minimum of λ: $d\sin\theta = m\lambda + \frac{\lambda}{N}$ --- (68)

Maximum of $\lambda + \Delta\lambda$: $d\sin\theta = m(\lambda + \Delta\lambda)$ (67)

$\Rightarrow$ Resolving power: $\boxed{R = \frac{\lambda}{\Delta\lambda} = mN}$ —— (69)