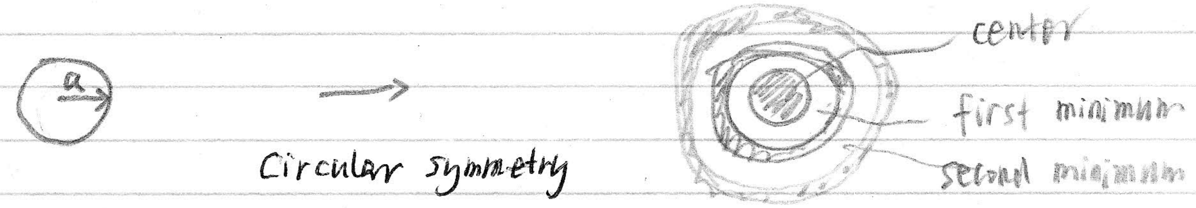
Lect. 19, Apr. 8, 2010

# * Diffraction by a circular aperture (Ch 18-3)

Aperture $r = a$ Diffraction Pattern



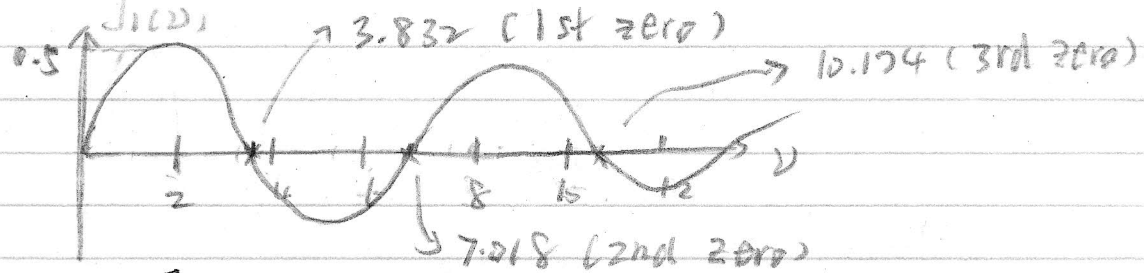

called "Airy Pattern"

Using two-dimensional Fourier transform.
Airy pattern can be derived as

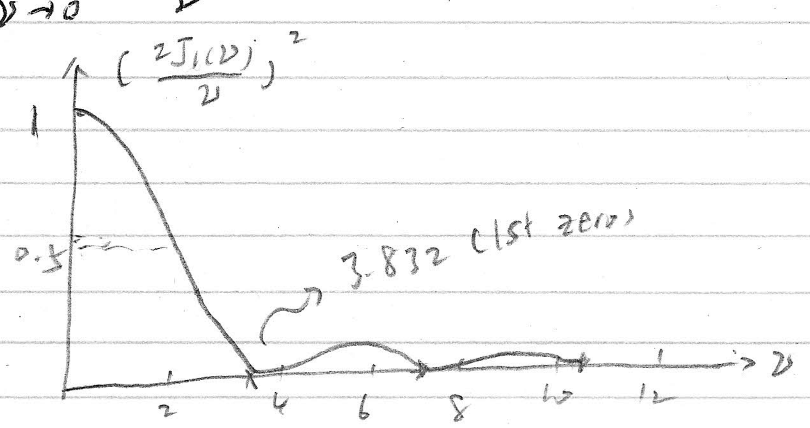
$$I = I_0 \left[ \frac{2J_1(\nu)}{\nu} \right]^2,$$

$\nu = \frac{2\pi}{\lambda} a \sin\theta$ : phase difference between the center and the edge of the circle for a given $\theta$

$J_1(\nu)$ : the Bessel function of the first order




$$\lim_{\nu \to 0} \frac{2J_1(\nu)}{\nu} = 1$$



\# (Continued)

First dark ring: $\nu = 3.832$

$$\sin\theta = \frac{\nu\lambda}{2\pi a} = \frac{\nu\lambda}{\pi D}$$

$$\sin\theta_1 = \frac{3.832}{\pi}\frac{\lambda}{D} = 1.22\frac{\lambda}{D} \quad \text{----} \quad (25)$$

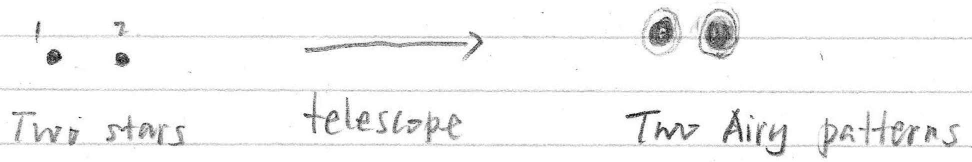
Note: mistake of 0.61 in the book

Thus, the size of the angular divergence:

$$\Delta\theta = \theta_1 = 1.22\frac{\lambda}{D}$$

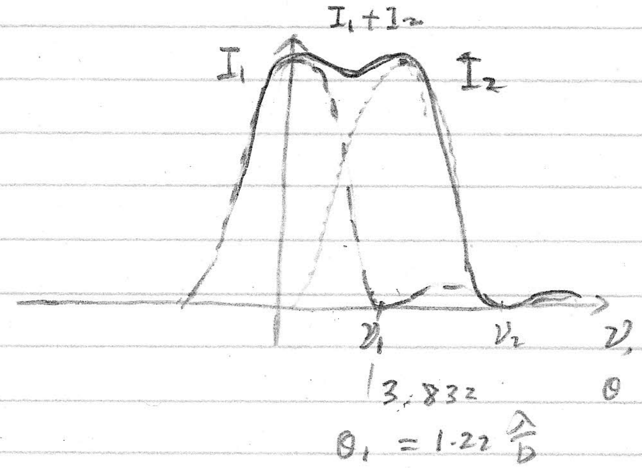
# Spatial resolution

e.g. telescope




To be just resolved: Rayleigh criterion.

The minimum of first spot coincides with the maximum of the second spot




Therefore, the spatial resolution

$$\Delta\theta = 1.22\frac{\lambda}{D}$$

Two point sources $> \Delta\theta$: can be resolved

$< \Delta\theta$: appear as a single source

Exp. Telescope $D = 80$ in, $\lambda = 6000\,\text{Å}$

$$\Delta\theta = 1.22\frac{\lambda}{D} = 1.22\cdot\frac{6.0\times10^{-7}\ \text{m}}{(80\ \text{in}\cdot 0.0254)} = 3.6\times10^{-7}\ \text{rad}$$

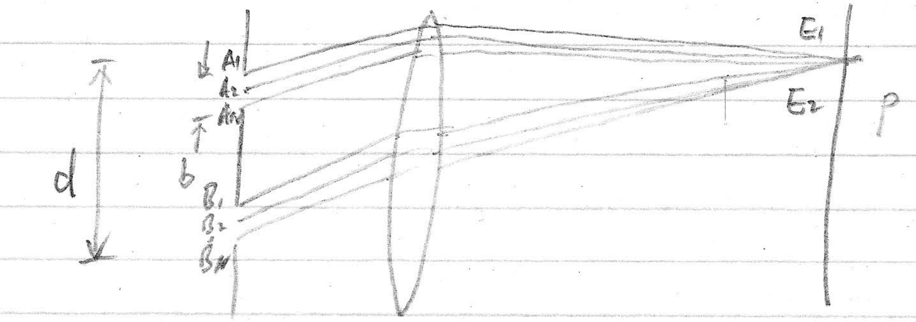
$$\Delta\theta = 3.6\times10^{-7}\ \text{rad}\cdot\frac{180}{\pi}(°/\text{rad})\cdot 3600\,('' / °) = 0.07''\ (\text{arcsec})$$

Exp. Eye: pupil $D = 2$ mm

$$\Delta\theta = 3\times10^{-4}\ \text{rad}$$

At $\underline{d = 20\ \text{m}}$, $l = d\cdot\Delta\theta = 6\times10^{-3}\ \text{m} \approx \underline{6\ \text{mm}}$. - for a good eyes.

# Two-slit Fraunhofer Diffraction Pattern (CH 18.6)




The combination of the single-slit diffraction and the double-slit interference.

$\Rightarrow$ The product of the two patterns

$$I = 4 I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\frac{\delta}{2}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda} = \frac{2\pi}{\lambda}\left(\underline{\frac{b}{2}\sin\theta}\right)$$

$$\delta = \frac{2\pi}{\lambda} \underline{d\sin\theta}$$

Prove:

$$E_1 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta)$$

$$E_2 = A\frac{\sin\beta}{\beta}\cos(\omega t - \beta - \Phi_1)$$

$$\Phi_1 = \frac{2\pi}{\lambda} d\sin\theta \quad (\text{the same as } \delta \text{ before})$$

phase difference from two slits

$$E = E_1 + E_2 = A\frac{\sin\beta}{\beta}\left[\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)\right]$$

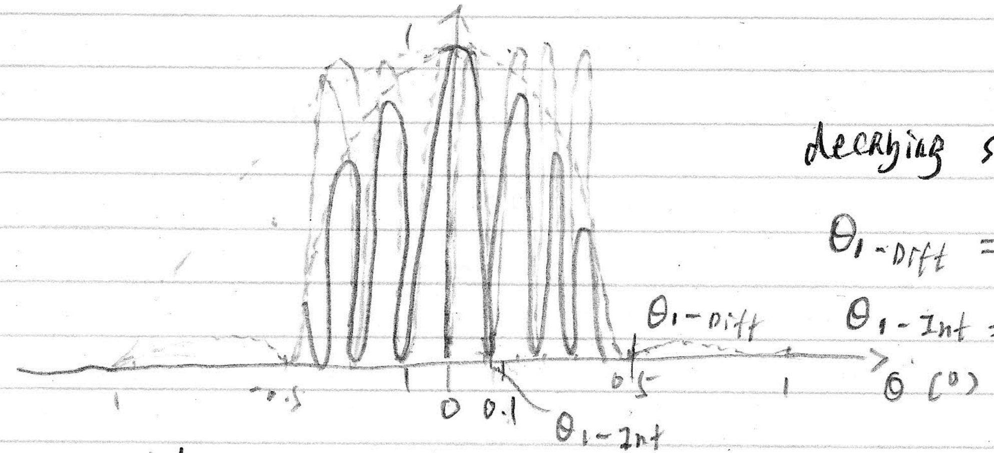
$$E = 2A\frac{\sin\beta}{\beta}\cos\frac{\Phi_1}{2}\cos\left(\omega t - \beta - \frac{\Phi_1}{2}\right)$$

$$\gamma = \frac{\Phi_1}{2} = \frac{\delta}{2}$$

$$I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma \quad \text{-------} \quad (45)$$

# Position of Maxima and Minima

since $d > b$, interference fringe width is smaller than the size of the diffraction




decaying sinusoidal function

$\theta_{1-Diff} = \frac{\lambda}{b}$

$\theta_{1-Int} = \frac{\lambda}{2d}$

Exp: $d = 0.035$ cm, $b = 0.0088$ cm, $\lambda = 6.328 \times 10^{-5}$ cm

Diffraction size $\theta_1 = \sin^{-1}\left(\frac{\lambda}{b}\right) = 0.00719$ rad

$\theta_1 = 0.412°$

Interference pattern maxima: $d\sin\theta = m\lambda \quad m = 0, \pm 1,$

minima: $d\sin\theta = (m+\frac{1}{2})\lambda, \ m = 0, 1 \cdots$

First minimum: $\sin\theta_{1I} = \frac{1}{2}\frac{\lambda}{d}$

$\theta_{1I} = 0.0009$ rad $= 0.05°$

Second minimum $\sin\theta_{2I} = \frac{3}{2}\frac{\lambda}{d} = 0.15°$

The width of interference fringe $= 0.1°$

# First minimum of diffraction: $\theta_1 = \frac{\lambda}{b}$

Fringe width of Interference $= \Delta\theta = \frac{\lambda}{d}$

Total number of Interference fringes within $(-\theta_1, \theta_1)$

$$\frac{\frac{\lambda}{b}}{\frac{\lambda}{d}} = \frac{d}{b}$$ $\Rightarrow$ multiple fringes in diffraction spot.