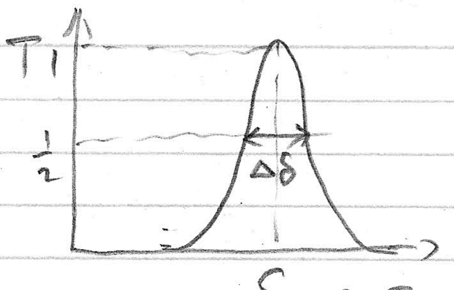
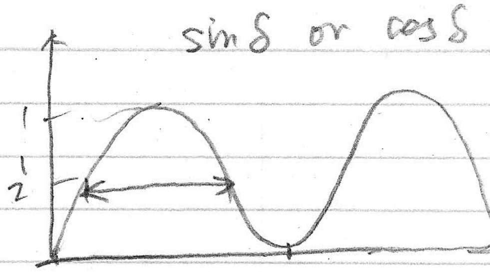
Lect. 17, April 1, 2010 ①

\# Find $\Delta\delta$ of FWHM (full width at half maximum)





$\delta_0 = 2m\pi$

FWHM $= \frac{2}{3}\pi$

At $\delta = \delta_0 = 2m\pi$, $T = 1$

At $\delta = \delta_0 + \frac{\Delta\delta}{2}$, $T = \frac{1}{2}$

Since $T = \dfrac{1}{1 + F\sin^2\frac{\delta}{2}}$

$$\frac{1}{2} = \frac{1}{1 + F\sin^2\left[\left(2m\pi + \frac{\Delta\delta}{2}\right)/2\right]}$$

$$F\sin^2\frac{\Delta\delta}{4} = 1$$

Since $\Delta\delta \ll 1$ $\Rightarrow$ $\left(\frac{\Delta\delta}{4}\right)^2 = \frac{1}{F}$

$$\Rightarrow \Delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}} \quad \text{---- (8)}$$

$R\uparrow$, $F\uparrow$, $\Delta\delta$ or FWHM decreases

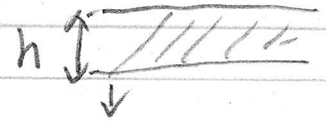
From one maximum to next maximum $\Delta\delta = 2\pi$

From the maximum to half of maximum $\Delta\delta = \frac{4}{\sqrt{F}}$

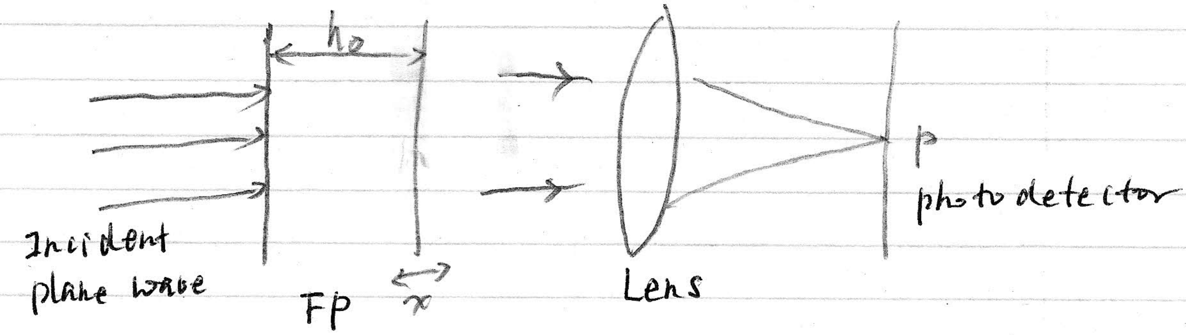
$F = 400$, $\Delta\delta = 0.2$

* Fabry - Perot Etalon : film h is fixed

* Fabry - Perot Interferometer : film h is adjustable.



Ⅱ Fabry - Perot Interferometer (16.4)



$$h = h_0 + x,$$

For normal incidence $\cos\theta_2 = 0$. air $n_2 = 1$

$$\delta = \frac{4\pi h}{\lambda_0} = \frac{4\pi \nu_0 (h_0 + x)}{c}$$

Exp: $h_0 = 10\,cm$, $\lambda_0 = 5000\,Å$, or $\nu_0 = 6\times10^{14}\,Hz$

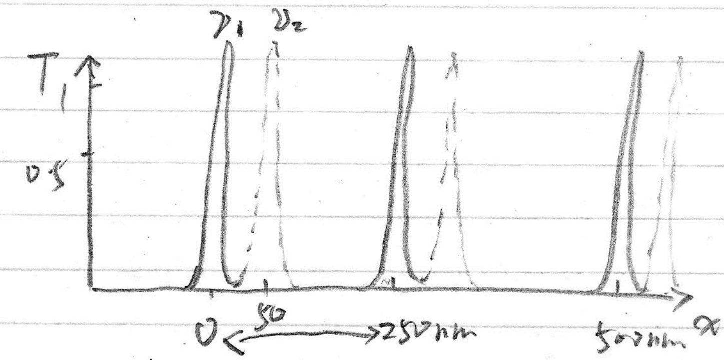
$$\delta = 800000\,\pi\left[1 + \frac{x}{h_0}\right]$$

The transmission resonances will occur for $\delta = 2m\pi$

$\delta = 800000\,\pi$, $800002\,\pi$, $800004\,\pi$

$m = 400000$ $\quad$ 400001 $\quad$ 400002

$x = 0$ $\quad$ $x = 250\,nm$ $\quad$ $x = 500\,nm$



$F = 1000$
Very sharp peak of transmission

transmission resonance

Two frequencies $\nu_1 = 6\times10^{14}\,Hz$ $\quad$ $\lambda_0 = 5000\,Å$

$\nu_2 = 6\times10^{14}\,Hz - 300\,MHz$; $\lambda_0 = 5000\,Å + 0.0025\,Å$

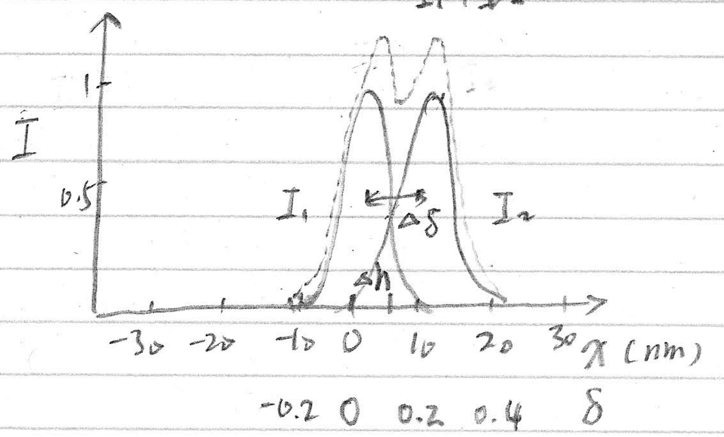
Same order $m = 400.000$, $\delta = 800,000\,\pi$

For ν_1, $x = 0$; $\quad$ For ν_2, $x = h_0 \frac{\Delta\nu}{\nu_0} = 5\times10^{-8}\,m = 50\,nm$

see next page

(Spectrum) Resolving Power of a (SCANNING) Fabry-Perot Interferometer

Rayleigh criterion: what is just resolved?



Just resolved: half intensity of I_1 falls on the half intensity of I_2.

$\Rightarrow$ The minimum of I_1+I_2 is about 74% of the maximum.

Half Intensity: $\delta = 2m\pi \pm \frac{\Delta\delta}{2}$. $\quad \Delta\delta$: FWHM, $\Delta\delta = \frac{4}{\sqrt{F}}$

For ν fixed: $\delta = \frac{4\pi h\nu}{c}$

$$\Delta\delta = \frac{4\pi\nu}{c}\Delta h \quad \Rightarrow \quad \Delta h = \frac{c}{\pi\nu}\frac{1}{\sqrt{F}}$$, shift Δh to get FWHM ~~half I~~

For two frequencies, ν_1, ν_2, what is $\Delta\nu = \nu_1 - \nu_2$ to have the same order of bright peaks when one plate is shifted by Δh?

$$\delta = 2m\pi = \frac{4\pi h_1\nu_1}{c} = \frac{4\pi(h_1+\Delta h)(\nu_1+\Delta\nu)}{c}$$

$$\left(1+\frac{\Delta h}{h_1}\right)\left(1+\frac{\Delta\nu}{\nu_1}\right) = 1 \quad \cancel{\Rightarrow \Delta\nu = \frac{\nu_1}{h_1}\Delta h}$$

$$\Rightarrow \boxed{\frac{\Delta\nu}{\nu_1} = -\frac{\Delta h}{h_1}} = \frac{c}{\pi\nu h_1\sqrt{F}}$$

Resolving Power $P = \left|\frac{\nu}{\Delta\nu}\right| = \frac{\pi h\nu\sqrt{F}}{c}$ ...... (22)

In term of λ_0, $P = \left|\frac{\lambda_0}{\Delta\lambda_0}\right| = \left|\frac{\nu_0}{\Delta\nu}\right| = \frac{\pi h\sqrt{F}}{\lambda_0}$ ...... (23)

Resolving Power (continued)

Exp: $P \propto h$, $\quad P \propto \sqrt{F}$

$h = 1\ cm$, $\quad \lambda_0 = 6 \times 10^{-5}\ cm = 600\ nm$, $\quad \nu_0 = 5 \times 10^{14}\ Hz$

$$P = \frac{\pi \cdot 10^{-2} \cdot \sqrt{360}}{6 \times 10^{-7} (m)} = 9.9 \times 10^{5} \qquad F = 360$$

$$\Delta\lambda = \frac{\lambda_0}{P} = 6 \times 10^{-13}\ m = 6 \times 10^{-3}\ \mathring{A} = 0.006\ \mathring{A} = 0.0006\ nm$$

$$\Delta\nu = \frac{\nu_0}{P} = \frac{5 \times 10^{14}\ Hz}{9.9 \times 10^{5}} = 5 \times 10^{8}\ Hz = 500\ MHz$$

As a comparison:

spectrum resolution of prism: $\sim 1\ \mathring{A}$

" of grating: $\sim 0.1\ \mathring{A}$

" of Fabry-Perot $\sim 0.01\ \mathring{A}$