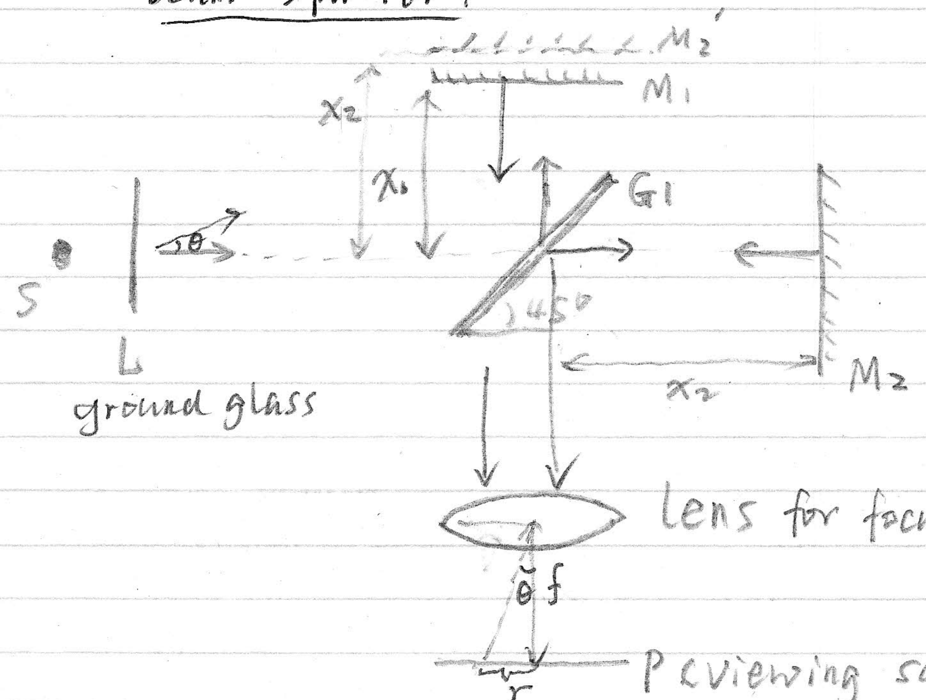
Lect. 16, March 30, 2010 

# Michelson **Interferometer** (CH 15.11)

→ Accurately measure the change of distance

two beams come from two mirrors that perpendicular to **each** other. The two mirrors receive light beams from the same source, thus coherent, through a beam splitter.




The light reflected by the splitter $G_1$ toward the viewing screen has an abrupt phase change of $\pi$

Optical path difference $\Delta = 2(x_2 - x_1)\cos\theta = 2d\cos\theta$

constructive $\Delta = (m + \frac{1}{2})\lambda$, $m = 0, 1, 2$

Destructive $\Delta = m\lambda$, $m = 0, 1, 2.$

$d$: the distance between mirror $M_1$ and $M_1'$

$\theta$: the angle the rays make with the axis

# Interference fringe patter: concentric dark and bright rings

Exp, $\lambda = 6 \times 10^{-5}$ cm, $d = 0.3$ mm.

the angles of the dark ring: $2d\cos\theta = m\lambda$

$$\theta = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left(\frac{m}{1000}\right)$$

$\theta = 0$ the center, $m = 1000$, the 1000th dark ring

# Michelson Interferometer (CH 15.11) – continued

$m = 1000$, dark ~~ring~~ at center, $\theta = 0$
$m = 999$, $\theta = 2.56°$
$m = 998$, $\theta = 3.62°$
$m = 997$, $\theta = 4.44°$

IF decreasing $d$: ① fringe rings collapse toward the center.
rings in view are lower order
② the spacing between rings increase

exp. $d = 0.15$ mm
$$\theta = \cos^{-1}\left(\frac{m\lambda}{2d}\right) = \cos^{-1}\left(\frac{m}{500}\right)$$

$m = 500$, dark fringe at center, $\theta = 0$
$m = 499$, $\theta = 3.62°$
$m = 498$, $\theta = 5.13°$
$m = 497$, $\theta = 6.28°$

# Distance from the center: $r = f\sin\theta$ ; $f$: focal length

# Michelson – Morley Experiment
(CH 30.10)

Prove that "ether" doesn't exist.
⇒ Light speed is constant in free space, independent of the inertial frames of coordinates.

# Meter: defined by Michelson
$\lambda = 6438.4696\ \text{Å}$. a Cadmium line
$1m = 1,553,164.13$ red Cadmium ~~the~~ wavelengths

# Multiple-Beam Interferometry (CH16)

* Application: Fabry-Perot Interferometer
* Resolving Power of Spectrum

The intensity interference pattern of $\boxed{\text{two beams}}$.
either due to split of ~~phase~~ wave front, or amplitude

In general $I = 4I_0 \cos^2\frac{\delta}{2}$

$$\delta = \frac{2\pi}{\lambda}\cdot\Delta.$$

$\delta$: phase difference  $\Delta$: optical path difference

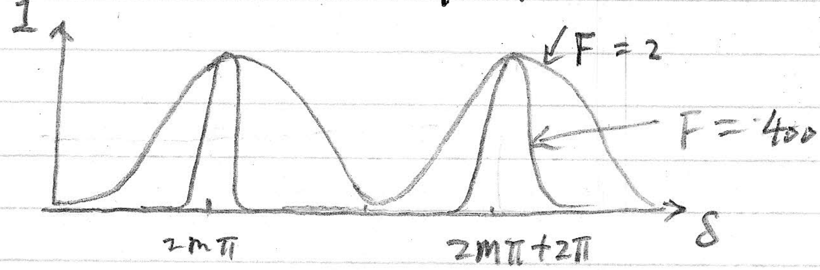
If one beam has abrupt phase change of $\pi$, due to reflection

$$I = 4I_0 \sin^2\frac{\delta}{2}$$

In the case of multiple beam interference

$$I \propto I_0 \frac{1}{1+F\sin^2\frac{\delta}{2}}$$

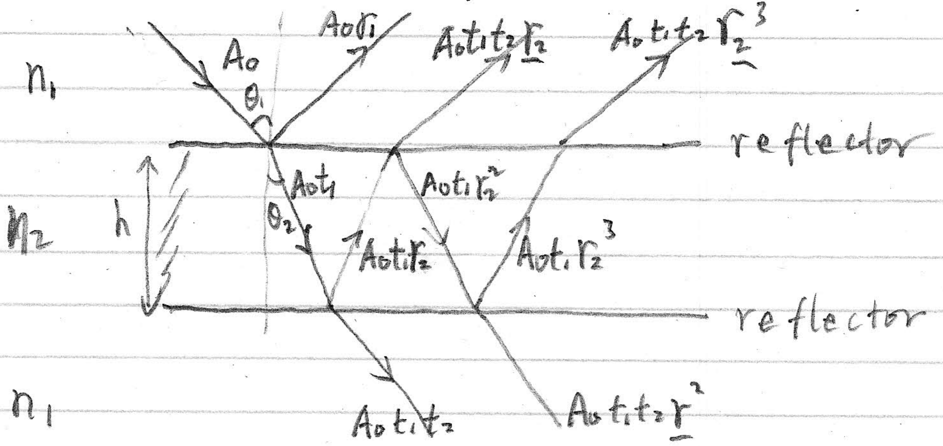
F: coefficient of Finesse




The higher the F, the more concentration of the light

Very low value of F ⇒ reduce to two beams.
⇒ sinusoidal variation of fringe

# multiple beam interference from a plane parallel film




Fabry-Perot Etalon

The reflected wave

1st : $A_0 r_1$

2nd : $A_0 t_1 t_2 r_2 \cdot e^{i\delta}$

3rd : $A_0 t_1 t_2 r_2^3 \, e^{i2\delta}$

4th : $A_0 t_1 t_2 r_2^4 \cdot e^{i3\delta}$

⋮

$$\delta = \frac{2\pi}{\lambda_0}\Delta = \frac{2\pi}{\lambda_0}\cdot \underbrace{2n_2 h\cos\theta_2}_{\text{cosine law}} = \frac{4\pi n_2 h\cos\theta_2}{\lambda_0} \quad (1)$$

The interference fringe

$$A_r = A_0[r_1 + t_1 t_2 r_2 e^{i\delta}(1 + r_2^2 e^{i\delta} + r_2^4 e^{i2\delta} + \cdots)]$$

Using Taylor series : $\frac{1}{1-x} = 1 + x + x^2 + x^3 \cdots$

$$A_r = A_0[r_1 + \frac{t_1 t_2 r_2 e^{i\delta}}{1 - r_2^2 e^{i\delta}}] \quad \text{--- (2)}$$

The definition : Reflectivity $R = r_1^2 = r_2^2$, $r_2 = -r_1$

At the interface : Transmittivity $\tau = t_1 t_2 = 1 - R$

The reflectivity of the Fabry-Perot Etalon

$$\mathcal{R} = \left|\frac{A_r}{A_0}\right|^2 = \left|r_1\left(1 - \frac{(1-R)e^{i\delta}}{1 - R^2 e^{i\delta}}\right)\right|^2$$

$$\mathcal{R} = R\left|\frac{1 - e^{i\delta}}{1 - Re^{i\delta}}\right|^2$$

# Multiple beam interference (continued)

$$1 - e^{i\delta} = (1-\cos\delta) - i\sin\delta$$

$$|1-e^{i\delta}|^2 = (1-\cos\delta)^2 + \sin^2\delta = 1 - 2\cos\delta + \cos^2\delta + \sin^2\delta$$

$$= 2(1-\cos\delta) = 4\sin^2\frac{\delta}{2}$$

$$\mathcal{R} = \frac{4R\sin^2\frac{\delta}{2}}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}} = \frac{\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

$$\mathcal{R} = \frac{F\sin^2\frac{\delta}{2}}{1+F\sin^2\frac{\delta}{2}} \quad \text{------} \quad (3)$$

$$F = \frac{4R}{(1-R)^2} \quad \text{------} \quad (4)$$

$F$: the coefficient of Finesse

Exp. $R = 0.9 \Rightarrow F = \frac{4 \cdot 0.9}{0.1^2} = 360$

Similarly, for transmission

1st: $A_0 t_1 t_2$

2nd: $A_0 t_1 t_2 r_2^2 e^{i\delta}$

3rd: $A_0 t_1 t_2 r_2^4 e^{i2\delta}$

4th: $A_0 t_1 t_2 r_2^6 e^{i3\delta}$

$$A_t = A_0 t_1 t_2 (1 + r_2^2 e^{i\delta} + r_2^4 e^{i2\delta} + r_2^6 e^{i3\delta} + \cdots)$$

$$A_t = A_0 \frac{t_1 t_2}{1 - r_2^2 e^{i\delta}} = A_0 \frac{1-R}{1-Re^{i\delta}}$$

The transmittivity of the Fabry-Perot Etalon $T$.

$$T = \left|\frac{A_t}{A_0}\right|^2 = \frac{(1-R)^2}{(1-R\cos\delta)^2 + R^2\sin^2\delta}$$

$$T = \frac{1}{1+F\sin^2\frac{\delta}{2}} \quad \text{------} \quad (5)$$

If $\delta = 2m\pi$, $m = 1, 2, \cdots$ or $\Delta = m\lambda_0$

$\Rightarrow T = 1$, complete transmission