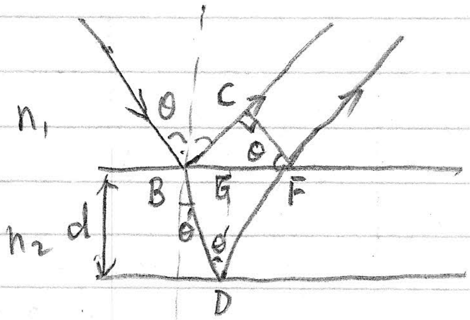
Lect. 15, March 25, 2010 ①

Oblique Incidence on a film — the Cosine Law (CH 15-3)

The difference of the optical paths



$\Delta = 2n_2 d\cos\theta'$

θ': angle of refraction

$\Delta = n_2(BD + DF) - n_1 BC$

Consider ΔBDG. $BD = DF = \dfrac{DG}{\cos\theta'} = \dfrac{d}{\cos\theta'}$

$BF = 2BG = 2\cdot DG\cdot\tan\theta' = 2d\tan\theta'$

Consider ΔBFC. $BC = BF\cdot\sin\theta = BF\cdot\sin\theta'\dfrac{n_2}{n_1}$

$BC = 2d\dfrac{\sin^2\theta'}{\cos\theta'}$

$\Delta = 2n_2 d\left(\dfrac{1}{\cos\theta'} - \dfrac{\sin^2\theta'}{\cos\theta'}\right)$

$\Delta = 2n_2 d\dfrac{\cos^2\theta'}{\cos\theta'} = 2n_2 d\cos\theta'$

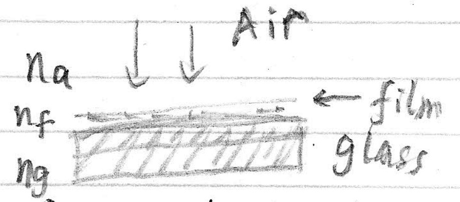
Δ: depends on θ, and also depend on λ_0 $(\delta = \frac{2\pi}{\lambda_0}\Delta)$

$\Rightarrow$ Explain the colors on soap bubbles

* Non reflecting Films (CH 15.4)

Example: anti reflective coating of eye class

Thickness of the film d



Looking for destructive interference

Since $n_g > n_f > n_a$, phase changes of π for both reflection

$$\Delta = \frac{1}{2}\lambda_0 = 2 n_f d$$

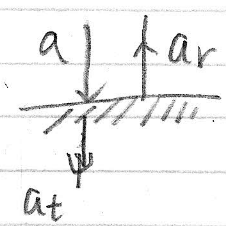
$$\Rightarrow \quad d = \frac{\lambda_0}{4 n_f}$$

Exp: $\lambda_0 = 5 \times 10^{-5}$ nm, $n_f = 1.38$ (MgF_2 film)

$$d = 0.9 \times 10^{-5} \text{ cm}$$

* Reflectivity R. reflection coefficient r

transmission coeffient t



$a_r = ar$

$a_t = at$

a, a_r, a_t are amplitudes of incident, reflected, transmitted light beams

From electromagnetic theory (CH 24.2, Eq. 67-72)

$$\left.\begin{aligned} r &= \frac{n_1 - n_2}{n_1 + n_2} \\ t &= \frac{2n_1}{n_1 + n_2} \end{aligned}\right\} \text{ from medium } n_1 \text{ to } n_2$$

Reflectivity $R = r^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

For eyeglass without coating.

$n_1 = 1$, $n_2 = 1.5$

$R = 0.04$, or 4% reflection of light

Continue on Non reflective Films (CH 15.4)

With coating, destructive interference

$$a_1 = \frac{n_f - n_a}{n_f + n_a} a \quad \text{wave reflected from upper surface}$$

$$a_2 = \frac{n_g - n_f}{n_g + n_a} a \quad \text{wave reflected from lower surface}$$

$$a_R = a_1 - a_2$$

$$R = \left(\frac{a_R}{a}\right)^2 = \left(\frac{n_f - n_a}{n_f + n_a} - \frac{n_g - n_f}{n_g + n_f}\right)^2$$

$n_a = 1$, $n_f = 1.38$, $n_g = 1.5$

$\Rightarrow R = 1.3\%$

Minimam happens when $a_1 = a_2 \Rightarrow n_f = \sqrt{n_a n_g}$

High Reflectivity Film (CH 15.5)

$$r = \frac{n_2 - n_1}{n_2 + n_1}$$

choose large n_2, e.g. $n_2 = 2.37$ (Zinc sulfide film)

$n_a = 1$, $n_f = 2.37$, $n_g = 1.5$

Since $n_a < n_f > n_g$

Constructive interference occurs at

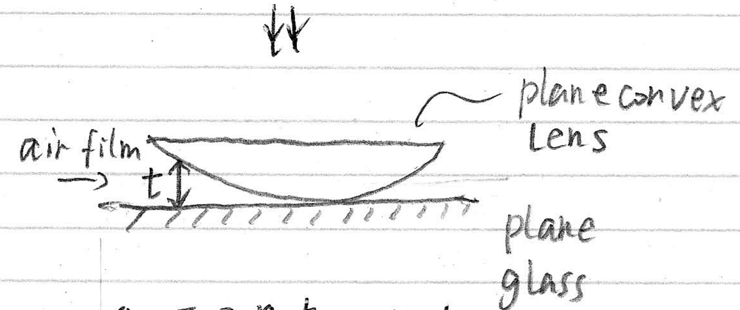
$$\Delta = \frac{1}{2}\lambda_0 = 2 n_f d$$

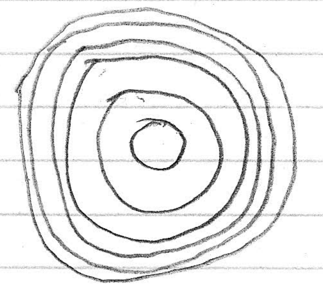
$$d = \frac{\lambda_0}{4 n_f}$$

$R \approx 35\%$

Exp: sunglass

Newton's Rings (CH 15.10)





Newton's Ring
concentric dark and bright rings

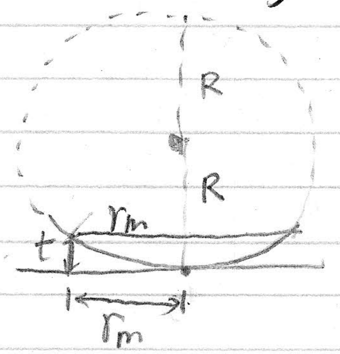
$\Delta = 2nt$, $n=1$

maxima : $\Delta = (m+\frac{1}{2})\lambda$, $m=0,1.$

minima $\Delta = m\lambda$, $m=0,1.$

Derive t:

using the identity of bisector



$r_m \cdot r_m = t \cdot (2R)$

$r_m^2 = 2Rt$

For ~~maxima~~ mth dark ring, $r_m^2 = 2R \cdot \frac{m\lambda}{2}$

$$r_m^2 = m\lambda R \qquad m=0,1,2\ldots$$

Exp. $\lambda = 6\times10^{-5}$ cm and $R = 100$ cm

$$r_m = \sqrt{m\lambda R} = 0.08\sqrt{m} \text{ cm}$$

The wavelength λ is amplified by $\sqrt{R}$

To find wavelength, counter the ring and measure r_m

Exp. from mth ring to (m+P)th ring

r_m $\qquad$ r_{m+p} $\qquad$ eg. $P=10$

$$\lambda = \frac{r_{m+p}^2 - r_m^2}{PR}$$