Lect. 13, March 18, 2010 ①

#1 Intensity Distribution of Light (CH 14.6)

Light is characterized by its electric field
Assuming $\vec{E}$ only along $\hat{i}$ (or x) direction

$$\vec{E_1} = \hat{i}\, E_{01} \cos\left(\frac{2\pi}{\lambda}\overline{S_1 P} - \omega t\right)$$

$$\vec{E_2} = \hat{i}\, E_{02} \cos\left(\frac{2\pi}{\lambda}\overline{S_2 P} - \omega t\right)$$

Intensity of light $I_{(t)} = K|\vec{E}_{(t)}|^2$, $K = \varepsilon_0 c^2$
For the measurement of intensity, it is averaged over time

$$I = \langle I_{(t)} \rangle \quad ; \quad \langle f_{(t)} \rangle \text{ average} = \frac{1}{2\tau}\int_{-\tau}^{\tau} f_{(t)}\, dt$$

$$\langle \cos^2(kx-\omega t)\rangle = \frac{1}{2\tau}\int_{-\tau}^{\tau} \cos^2(kx-\omega t)\, dt$$

$$= \frac{1}{2\tau}\int_{-\tau}^{\tau} \frac{1+\cos[2(kx-\omega t)]}{2}\, dt$$

$$= \frac{1}{\cancel{2\tau}}\cdot\frac{1}{\cancel{2}}\, t\Big|_{-\tau}^{\tau} + \frac{2}{4\tau}\int_{0}^{\tau} \cos[2(kx-\omega t)]\, dt$$

$$= \frac{1}{2} - \frac{1}{2\tau}\cdot\frac{1}{2\omega}\sin[2(kx-\omega t)]\Big|_{0}^{\tau}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\tau} = \frac{\pi}{\tau}$$

$$\sin\left[2kx - 2\frac{\pi}{\tau}\cdot\tau\right]\Big|_{t=\tau} = \sin 2kx$$

$$\sin[2kx - \omega t]\Big|_{t=0} = \sin 2kx$$

$$\therefore \langle \cos^2(kx-\omega t)\rangle = \frac{1}{2}$$

$$\therefore I = \frac{1}{2} K E_0^2$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$I = |\vec{E}|^2 = |\vec{E_1}|^2 + |E_2|^2 + 2\vec{E_1}\cdot\vec{E_2}$$

$$\langle I \rangle = k\Big[E_{01}^2 \overbrace{\cancel{\cos^2\left(\tfrac{2\pi}{\lambda}\overline{S_1P} - \omega t\right)}}^{\frac{1}{2}} + E_{02}^2 \overbrace{\cancel{\cos^2\left(\tfrac{2\pi}{\lambda}\overline{S_2P} - \omega t\right)}}^{\frac{1}{2}}$$

$$+ E_{01}E_{02}\left\{ \cos\left[\tfrac{2\pi}{\lambda}(\overline{S_2P} - \overline{S_1P})\right] + \underbrace{\cos\left[2\omega t - \tfrac{2\pi}{\lambda}(\overline{S_2P} + \overline{S_1P})\right]}_{0} \right\}\Big]$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$\delta = \frac{2\pi}{\lambda}(\overline{S_2P} - \overline{S_1P})$: the phase difference caused by the two paths. $\Delta = \overline{S_2P} - \overline{S_1P}$

$$\delta = \frac{2\pi}{\lambda}\cdot\Delta$$

$\delta = 2n\pi \Rightarrow$ constructive interference. $\cos\delta = 1$

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$\delta = (2n+1)\pi \Rightarrow$ destructive interference $\cos\delta = 1$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If the ~~so~~ two light sources are incoherent,

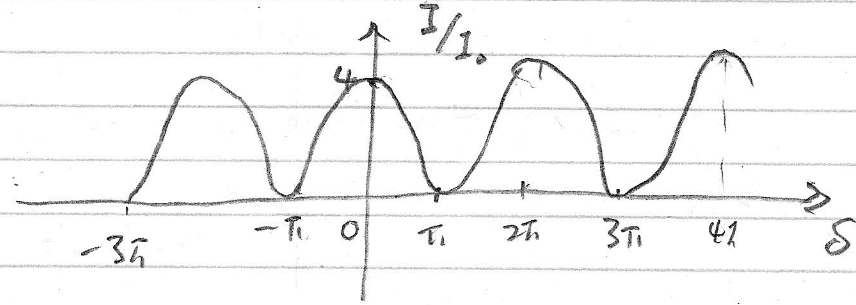
δ is not a constant, but random

$\langle \cos\delta \rangle = 0$

$I = I_1 + I_2$: simple addition without interference

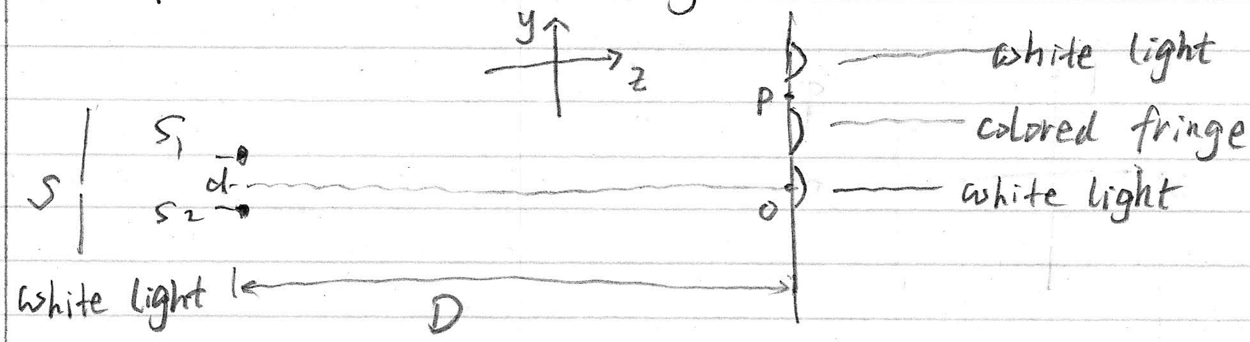
If $I_1 = I_2 = I_0$

$$I = 2I_0 + 2I_0\cos\delta = 4I_0\cos^2\frac{\delta}{2}$$



Intensity Variation with phase difference

Interference with White Light (CH 14.9)



White light: 4000 Å — 7000 Å (4×10^{-5} cm — 7×10^{-5} cm)
Red — Violet

At 0: $\Delta = 0$, $\delta = \frac{2\pi}{\lambda}\Delta = 0$ for all wavelength
$\Rightarrow$ white light

At $y > 0$, $\Delta = \frac{d}{D}y$; $\delta = \frac{2\pi}{\lambda}\Delta = \frac{d}{D}\cdot y \cdot \frac{2\pi}{\lambda}$

$$y = \frac{D}{d}\cdot\frac{\delta}{2\pi}\cdot\lambda$$

For first order fringes: $\delta = 2\pi n$ at $n=1$, or 2π

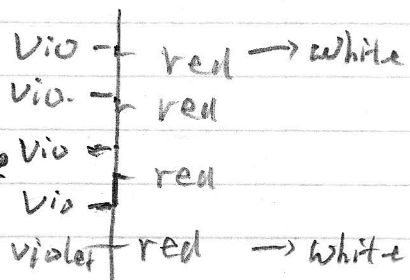
$$y = \frac{D}{d}\lambda \Rightarrow y \propto \lambda.$$

red lies far from center, and seperate from violet

However, as y increases, lights become mixed again

Exp. $y = 30\times10^{-2}$ cm, $\frac{D}{d} = 1000$

$$\Delta = \frac{d}{D}y = 30\times10^{-5}\text{ cm}$$



Wavelengths λ with constructive interference

$$\lambda = \frac{\Delta}{n} = \frac{30\times10^{-5}\text{ cm}}{n}$$

$n=4$, $\lambda = 7.5\times10^{-5}$ cm red
$n=5$ $\lambda = 6.0\times10^{-5}$ cm yellow
$n=6$, $\lambda \approx 5\times10^{-5}$ cm green-yellow
$n=7$. $\lambda = 4.3\times10^{-5}$ cm, Violet

$\Rightarrow$ white