

Lect. 13, March 18, 2010

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# Intensity Distribution of Light (CH 14.6)

Light is characterized by its electric field  
Assuming  $\vec{E}$  only along  $\hat{i}$  (or x) direction

$$\vec{E}_1 = \hat{i} E_{01} \cos\left(\frac{2\pi}{\lambda} \bar{S}_1 P - \omega t\right)$$

$$\vec{E}_2 = \hat{i} E_{02} \cos\left(\frac{2\pi}{\lambda} \bar{S}_2 P - \omega t\right)$$

Intensity of light  $I_{\text{eff}} = k |E_{\text{eff}}|^2$ ,  $k = \epsilon_0 c^2$

For the measurement of intensity, it is averaged over time

$$I = \langle I_{\text{eff}} \rangle ; \langle f_{\text{eff}} \rangle \text{ average} = \frac{1}{2\tau} \int_{-\tau}^{\tau} f_{\text{eff}}(t) dt$$

$$\langle \cos^2(kx - \omega t) \rangle = \frac{1}{2\tau} \int_{-\tau}^{\tau} \cos^2(kx - \omega t) dt$$

$$= \frac{1}{2\tau} \int_{-\tau}^{\tau} \frac{1 + \cos[2(kx - \omega t)]}{2} dt$$

$$= \frac{1}{2\tau} \cdot \frac{1}{2} t \Big|_{-\tau}^{\tau} + \frac{2}{4\tau} \int_0^{\tau} \cos[2(kx - \omega t)] dt$$

$$= \frac{1}{2} - \frac{1}{2\tau} - \frac{1}{2\omega} \sin[2(kx - \omega t)] \Big|_0^{\tau}$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\tau} = \frac{\pi}{\tau}$$

$$\sin\left[2kx - 2\frac{\pi}{\tau} \cdot \tau\right] \Big|_{t=\tau} = \sin 2kx$$

$$\sin[2kx - \omega t] \Big|_{t=0} = \sin 2kx$$

$$\therefore \langle \cos^2(kx - \omega t) \rangle = \frac{1}{2}$$

$$\therefore I = \frac{1}{2} k E_0^2$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$I = |\vec{E}|^2 = |\vec{E}_1|^2 + |\vec{E}_2|^2 + 2\vec{E}_1 \cdot \vec{E}_2$$

$$\langle I \rangle = K \left[ E_{01}^2 \cos^2 \left( \frac{2\pi}{\lambda} \bar{s}_1 p - \omega t \right) + E_{02}^2 \cos^2 \left( \frac{2\pi}{\lambda} \bar{s}_2 p - \omega t \right) + E_{01} E_{02} \left\{ \cos \left[ \frac{2\pi}{\lambda} (\bar{s}_2 p - \bar{s}_1 p) \right] + \cos \left[ 2\omega t - \frac{2\pi}{\lambda} (\bar{s}_2 p + \bar{s}_1 p) \right] \right\} \right] \quad (2)$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$\delta = \frac{2\pi}{\lambda} (\bar{s}_2 p - \bar{s}_1 p)$ ; the phase difference caused by the two paths;  $\Delta = \bar{s}_2 p - \bar{s}_1 p$

$$\delta = \frac{2\pi}{\lambda} \cdot \Delta$$

$\delta = 2n\pi \Rightarrow$  constructive interference.  $\cos \delta = 1$

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$\delta = (2n+1)\pi \Rightarrow$  destructive interference  $\cos \delta = -1$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

If the ~~two~~ two light sources are incoherent,

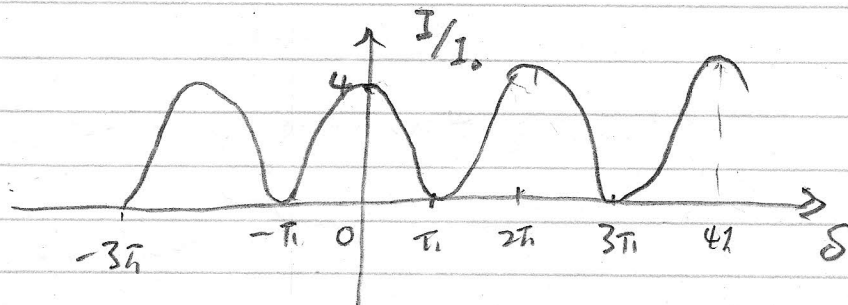
$\delta$  is not a constant, but random

$$\langle \cos \delta \rangle = 0$$

$I = I_1 + I_2$ : simple addition without interference

If  $I_1 = I_2 = I_0$

$$I = 2I_0 + 2I_0 \cos \delta = 4I_0 \cos^2 \frac{\delta}{2}$$



Intensity Variation with phase difference

