

Lect. 12 March 16, 2010

①

# Graphical Representation of Superposition: vector method

$$y_1 = a_1 \cos(\omega t + \theta_1)$$

$$y_2 = a_2 \cos(\omega t + \theta_2)$$

$$y = a \cos(\omega t + \theta)$$

Use the sum of vectors

$\vec{OA}_1 = (a_1, \theta_1)$  in polar coordinate,  
or in  $(a, \theta)$  parameter space

$$\vec{OA}_2 = (a_2, \theta_2)$$

$$\vec{OA} = (a, \theta)$$

$$\vec{OA} = \vec{OA}_1 + \vec{OA}_2$$

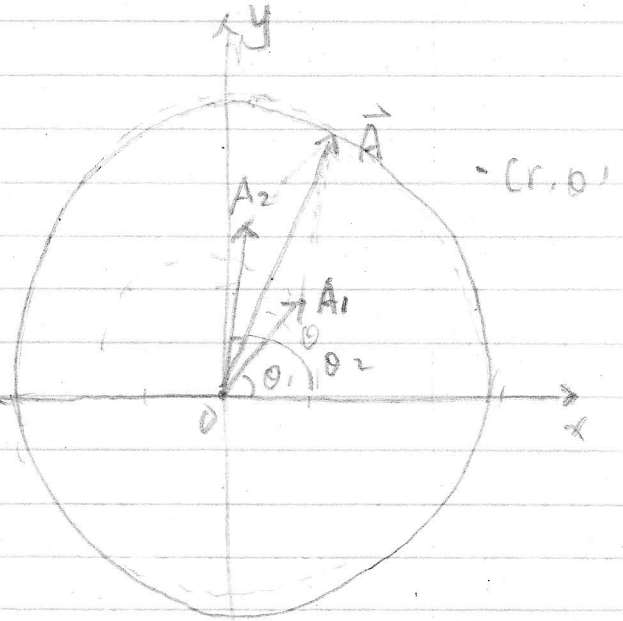
It is obvious that

projection on x

$$\Rightarrow a \cos \theta = a_1 \cos \theta_1 + a_2 \cos \theta_2$$

Projection on y

$$\Rightarrow a \sin \theta = a_1 \sin \theta_1 + a_2 \sin \theta_2$$



$$\Rightarrow a = [a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2)]^{\frac{1}{2}} \quad \text{The whole pattern of } \vec{OA}_1 \vec{OA}_2 \text{ rotates at } \omega$$

If  $a_1 = a_2$ ,  $\phi = \theta_1 - \theta_2$  = phase difference;

$$a = [2a^2 (1 + \cos \phi)]^{\frac{1}{2}} = [2a^2 \cdot 2 \cos^2 \frac{\phi}{2}]^{\frac{1}{2}}$$

$$a = 2a_1 \cos^2 \frac{\theta}{2} \quad \Rightarrow \begin{cases} a = 2a_1, & \theta = 0, 2\pi, 2\pi n \\ a = 0, & \theta = \pi, (\pi + \frac{1}{2}) \cdot 2\pi \dots \end{cases}$$

### Complex Representation of Superposition

$$y_1 = a_1 \cos(\omega t + \theta_1)$$

$$\text{or } y_1 = a_1 e^{i(\omega t + \theta_1)}$$

the displacement =  $\text{Re}(y_1)$

$$y_2 = a_2 e^{i(\omega t + \theta_2)}$$

$$y = y_1 + y_2 = a_1 e^{i(\omega t + \theta_1)} + a_2 e^{i(\omega t + \theta_2)}$$

$$y = (a_1 e^{i\theta_1} + a_2 e^{i\theta_2}) e^{i\omega t}$$

$$y = a e^{i\theta} e^{i\omega t} = a e^{i(\omega t + \theta)}$$

$$\text{where } a e^{i\theta} = a_1 e^{i\theta_1} + a_2 e^{i\theta_2}$$

or	}	$a \cos \theta = a_1 \cos \theta_1 + a_2 \cos \theta_2$	Re part
		$a \sin \theta = a_1 \sin \theta_1 + a_2 \sin \theta_2$	Im part

(Not Covered) Consider  $N$  displacement with increasing phase  $\theta_0$   
e.g., diffraction grating step

$$y_1 = a e^{i\omega t}$$

$$y_2 = a e^{i(\omega t + \theta_0)}$$

$$y_3 = a e^{i(\omega t + 2\theta_0)}$$

$$\vdots$$
  
$$y_{N-1} = a e^{i(\omega t + (N-1)\theta_0)}$$

$$y = y_1 + y_2 + \dots + y_{N-1} = a e^{i\omega t} [1 + e^{i\theta_0} + e^{2i\theta_0} + \dots + e^{i(N-1)\theta_0}]$$

Using the formula of partial sum of series

$$1 + r + r^2 + \dots + r^{N-1} = \sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r} \quad (r \neq 1)$$

$$\lim_{N \rightarrow \infty} \sum_{k=0}^{N-1} r^k = \frac{1}{1-r}$$

