Lect. 12 March 16, 2010 ①

Graphical Representation of Superposition : vector method

$y_1 = a_1 \cos(\omega t + \theta_1)$

$y_2 = a_2 \cos(\omega t + \theta_2)$

$y = a \cos(\omega t + \theta)$

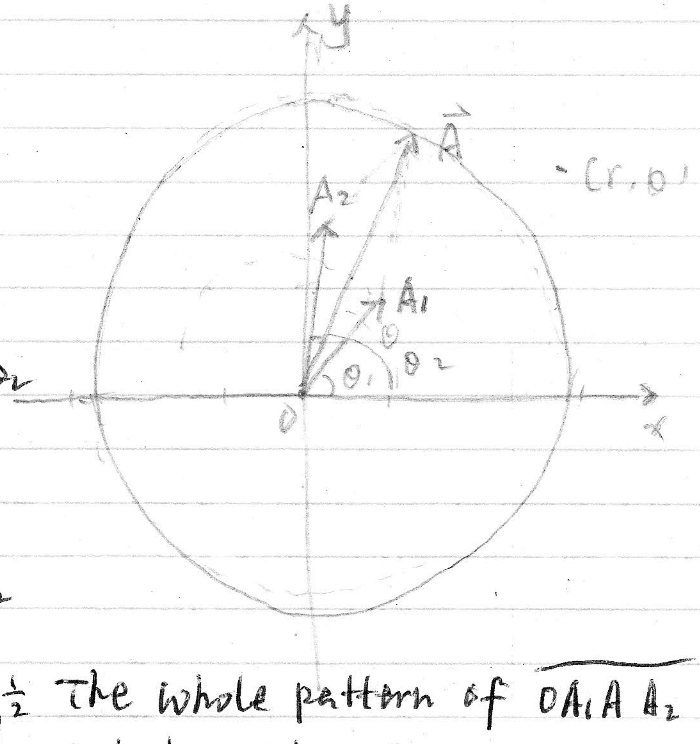
Use the sum of vectors

$\overrightarrow{OA_1} = (a_1, \theta_1)$ in polar coordinate, or in (a, θ) parameter space

$\overrightarrow{OA_2} = (a_2, \theta_2)$

$\overrightarrow{OA} = (a, \theta)$

$\overrightarrow{OA} = \overrightarrow{OA_1} + \overrightarrow{OA_2}$



It is obvious that

projection on x

$\Rightarrow a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2$

Projection on y

$\Rightarrow a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2$

$\Rightarrow a = [a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)]^{\frac{1}{2}}$

The whole pattern of $\overline{OA_1AA_2}$ rotates at ω

If $a_1 = a_2$, $\phi = \theta_1 - \theta_2$: phase difference:

$a = [2a_1^2(1 + \cos\theta)]^{\frac{1}{2}} = [2a^2 \cdot 2\cos^2\frac{\theta}{2}]^{\frac{1}{2}}$

$a = 2a_1\cos^2\frac{\theta}{2} \Rightarrow \begin{cases} a = 2a_1, & \theta = 0, 2\pi, 2\pi n \\ a = 0, & \theta = \pi, (n+\frac{1}{2}) \cdot 2\pi \cdots \end{cases}$

\# Complex Representation of Superposition

$$y_1 = a_1 \cos(\omega t + \theta_1)$$

or $y_1 = a_1 e^{i(\omega t + \theta_1)}$

the displacement $= \mathrm{Re}(y_1)$

$$y_2 = a_2 e^{i(\omega t + \theta_2)}$$

$$y = y_1 + y_2 = a_1 e^{i(\omega t + \theta_1)} + a_2 e^{i(\omega t + \theta_2)}$$

$$y = (a_1 e^{i\theta_1} + a_2 e^{i\theta_2}) e^{i\omega t}$$

$$y = a e^{i\theta} e^{i\omega t} = a e^{i(\omega t + \theta)}$$

where $a e^{i\theta} = a_1 e^{i\theta_1} + a_2 e^{i\theta_2}$

or $\begin{cases} a\cos\theta = a_1 \cos\theta_1 + a_2 \cos\theta_2 & \text{Re part} \\ a\sin\theta = a_1 \sin\theta_1 + a_2 \sin\theta_2 & \text{Im part} \end{cases}$

(Not Covered) # Consider N displacement with increasing phase θ_0 step
e.g., diffraction grating

$$y_1 = a e^{i\omega t}$$

$$y_2 = a e^{i(\omega t + \theta_0)}$$

$$y_3 = a e^{i(\omega t + 2\theta_0)}$$

$$\vdots$$

$$y_{N-1} = a e^{i[\omega t + (N-1)\theta_0]}$$

$$y = y_1 + y_2 + \cdots y_{N-1} = a e^{i\omega t}\left[1 + e^{i\theta_0} + e^{2i\theta_0} + \cdots e^{i(N-1)\theta_0}\right]$$

Using the formula of partial sum of series

$$1 + r + r^2 \cdots r^{N-1} = \sum_{k=0}^{N-1} r^k = \frac{1 - r^N}{1 - r} \qquad r \neq 1$$

$$\lim_{N \to \infty} \sum_{k=0}^{N-1} r^k = \frac{1}{1-r}$$

Two-Beam Interference: By Division of Wave Front (CH 14)
— Young's Double-hole Experiment

Interference Pattern of Water Surface Wave

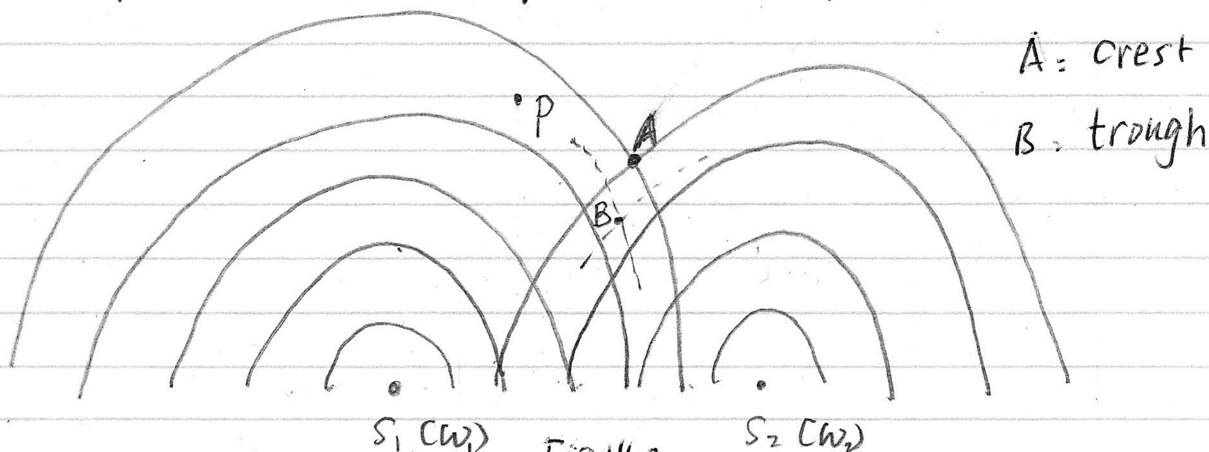


Fig 14.2

Forming a stationary interference pattern with crests & Troughs

$\omega_1 = \omega_2 = \omega$: coherent sources (S_1, S_2) constant phase difference

$y_1 = a_1 \cos(\omega t - k\,\overline{S_1P})$

$y_2 = a_2 \cos(\omega t - k\,\overline{S_2P})$

The phase difference $\Delta\phi = k(\overline{S_1P} - \overline{S_2P}) = \frac{2\pi}{\lambda}(\overline{S_1P} - \overline{S_2P})$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta$$

$\Delta = \overline{S_1P} - \overline{S_2P}$: difference of distance

1) $\Delta = n\lambda \Rightarrow \Delta\phi = 2\pi n \Rightarrow$ constructive interference

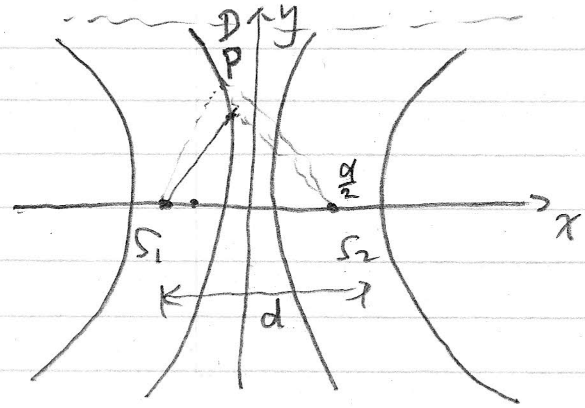
2) $\Delta = (n + \frac{1}{2})\lambda \Rightarrow \Delta\phi = 2\pi(n + \frac{1}{2}) \Rightarrow$ destructive interference

The locations of crests & troughs only depend on the distances ~~of~~ from the two sources $\Rightarrow$ stationary pattern

Nodal Lines:

the locus of points which correspond to minima

$\Delta = S_1P - S_2P = (n + \frac{1}{2})\lambda \Rightarrow$ hyperbola lines



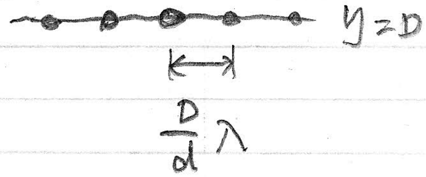


Fig 14.3

$S_1: (-\frac{d}{2}, 0)$, $\quad S_2: (\frac{d}{2}, 0) \qquad P(x, y)$

$$\overline{S_1P} = [(x + \frac{d}{2})^2 + y^2]^{\frac{1}{2}}$$

$$\overline{S_2P} = [(x - \frac{d}{2})^2 + y^2]^{\frac{1}{2}}$$

$$\Delta = \overline{S_1P} - \overline{S_2P}$$

Rearrange: $\quad \dfrac{x^2}{\frac{1}{4}\Delta^2} - \dfrac{y^2}{\frac{1}{4}(d^2 - \Delta^2)} = 1$

which is the standard equation of hyperbola

$\Delta = (n + \frac{1}{2})\lambda$: nodal lines

$\Delta = n\lambda$: maxima lines

For large (x, y), RHS $\to 0$

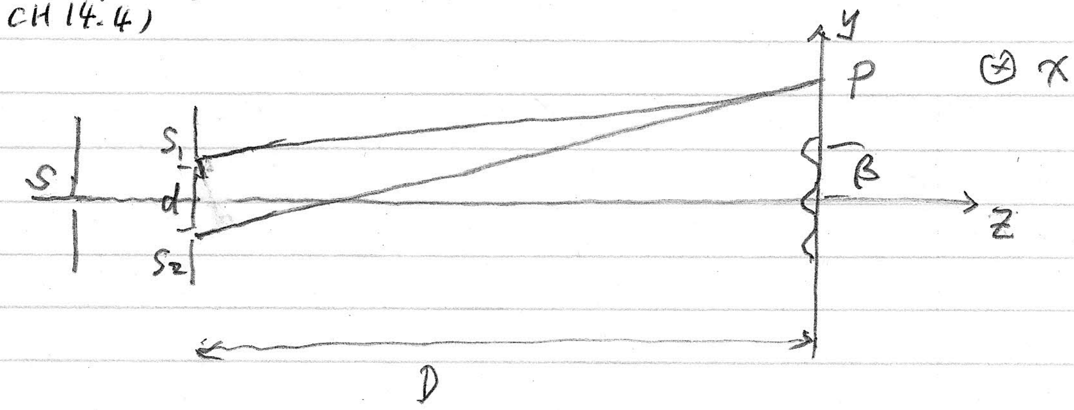
$$\frac{x^2}{\frac{1}{4}\Delta^2} - \frac{y^2}{\frac{1}{4}(d^2 - \Delta^2)} = 0$$

$y = \pm\left(\dfrac{d^2 - \Delta^2}{\Delta^2}\right)^{\frac{1}{2}} x$: turn into straight lines

$d \gg \Delta$: small phase difference ; $y = D \Rightarrow$ equal spacing

$y = \pm \frac{d}{\Delta} x = \pm \frac{d}{n\lambda} x \Rightarrow x = \pm n(\frac{D}{d})\lambda$ at $y = D$

★ Interference of Light: Young's Double-hole Experiment
(CH 14.4)



S is needed to make coherent lights at S_1 and S_2, since they emenates from the same source S

~~For maxi~~ $\Delta = S_2P - S_1P = f(d, D, y)$

$$(S_2P)^2 = D^2 + (y + \frac{d}{2})^2$$
$$(S_1P)^2 = D^2 + (y - \frac{d}{2})^2$$

$$(S_2P)^2 - (S_1P)^2 = 2yd$$

$$S_2P - S_1P = \frac{2yd}{(S_2P + S_1P)}$$

Assuming $y \ll D$, $d \ll D$ $\Rightarrow S_2P + S_1P = 2D$

$$\Delta = \frac{d}{D} y$$

or $$y = \frac{D}{d} \Delta$$

For maximu. $y_n = n\lambda \cdot \frac{D}{d}$ $(\Delta = n\lambda)$

β: the width of the fringe: distance between two consecutive bright fringes

$$\beta = \frac{\lambda D}{d}$$

From measurement of D, d and β $\Rightarrow \lambda$