Lect. 11, Mar. 4, 2010

Superposition of waves (CH13)

We have leaned that, in part II,

1. A periodic motion can be decomposed into a series of sinusoidal motion — Fourier series
2. A wave packet can be decomposed into a spectrum of sinusoidal waves — Fourier Integral

We are going to study, in part III & IV, on the other hand, the superposition of waves from multiple sources in space

part III: CH13 - 17: Interference

part IV: CH18 - 21: Diffraction

Inteference: superposition of waves from two or multiple point sources

e.g., Young's Interference Experiment (1801)

e.g., Michelson Interferometer (1881)

Diffraction: superposition of waves from one or multiple area sources. e.g., aperture

e.g., Diffraction by a circular Aperture

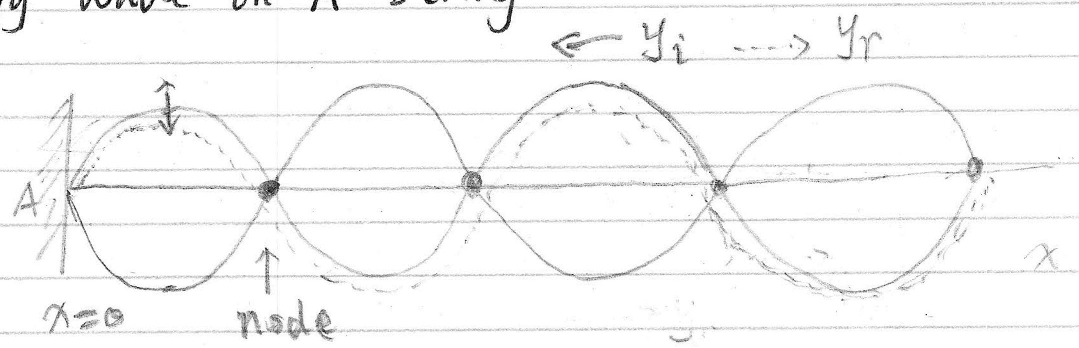
e.g., Diffraction Grating

→ spectrometer

Principle of Superposition: Linear combination

The resultant displacement (at a particular point) produced by a number of waves is the <u>vector sum</u> of the displacement produced by each one of the disturbances

Stationary wave on A string



Incident wave : $y_i(x,t) = a\sin(kx+\omega t \cancel{+\phi})$

The string is fixed at point A , $x=0$

$$y_i|_{x=0} = a\sin(\omega t)$$

There must be a ~~ϕ~~ reflected wave

$$y_r|_{x=0} = -y_i|_{x=0}, \text{ so } (y_r+y_i)|_{x=0} = 0$$

$$y_r|_{x=0} = -a\sin(\omega t) = a\sin(-\omega t)$$

The reflected wave

$$y_r = a\sin(kx-\omega t)$$

The resultant displacement is

$$y = y_i + y_r = a\sin(kx+\omega t) + a\sin(kx-\omega t)$$

$$= a[\sin kx\cos\omega t + \cancel{\cos kx \sin\omega t}$$
$$+ \sin kx\cos\omega t - \cancel{\cos kx \sin\omega t}]$$

$$y = 2a\sin kx \cdot \cos\omega t \Longrightarrow \text{stationary wave } V_g = 0$$

Node $y(x,t) = 0$ displament $= 0$ at all time

$$\sin kx = 0 \quad ; \quad \sin\frac{2\pi}{\lambda}x = 0 \quad ; \quad \frac{2\pi}{\lambda}x = n\pi$$

$$x = \frac{\lambda}{2}\cdot n \quad , \text{ or } \quad 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda.$$

Antinodes: displacement is maximum

$$\sin(\frac{2\pi}{\lambda}x) = 1$$

$$\frac{2\pi}{\lambda}x = n\cdot\frac{\pi}{2}, \quad n = 1, 3, 5.$$

$$x = \frac{\lambda}{4}\cdot n, \quad \text{or} \quad \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}$$

$$y = 2a\cos\omega t.$$

At antinodes, the amplitude of displacement is doubled.

String with two ends fixed $[0, L]$

$$y = 2a\sin kx\cdot\cos\omega t$$

Only certain frequencies are allowed → discrete λ_n, ω_n

$y|_{x=L} = 0$ all the time

$$\sin[\frac{2\pi}{\lambda}L] = 0$$

$$\frac{2\pi}{\lambda}L = n\pi \qquad n = 1, 2, \ldots$$

$$\lambda = \lambda_n = \frac{2L}{n}$$

$$\omega = \omega_n = v\cdot k_n = v\cdot\frac{2\pi}{\lambda_n} = \frac{v}{L}\pi n \quad n = 1, 2, \ldots$$

where $v = \sqrt{\frac{T}{\rho}}$ = a constant

phase differences

$$\Delta\phi = (kx + \omega t) - (kx - \omega t) = 2\omega t$$

is independent of x.

All the points along the string have the same phase.

General formula of Superposition of Two Sinusoidal wave

$$\cancel{y_1(x,t)} \; y_1(x_1,t) = a_1 \cos(\omega t - kx_1 + \theta_{10})$$

$$y_2(x_2,t) = a_2 \cos(\omega t - kx_2 + \theta_{20})$$

Two waves ① same frequency

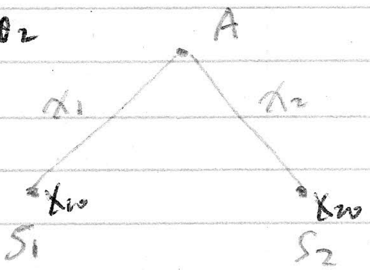
② from different sources, $x_{10} \neq x_{20}$

Consider at a particular point A

$y_1 = a_1 \cos(\omega t + \theta_1)$

$y_2 = a_2 \cos(\omega t + \theta_2)$

phase θ_1, θ_2 depends on distance



$y = y_1 + y_2$

$$y = a_1 \cos\omega t \cos\theta_1 - a_1 \sin\omega t \sin\theta_1 + a_2 \cos\omega t \cos\theta_2 - a_2 \sin\omega t \sin\theta_2$$

$$y = [a_1\cos\theta_1 + a_2\cos\theta_2]\cos\omega t - [a_1\sin\theta_1 + a_2\sin\theta_2]\sin\omega t$$

And: $y = a\cos(\omega t + \theta) = a\cos\theta\cos\omega t - a\sin\theta\sin\omega t$

$$a\cos\theta = a_1\cos\theta_1 + a_2\cos\theta_2 \quad ①$$

$$a\sin\theta = a_1\sin\theta_1 + a_2\sin\theta_2 \quad ②$$

$①^2 + ②^2$

$$\Rightarrow a = [a_1^2 + a_2^2 + 2a_1a_2\cos(\underline{\theta_1 - \theta_2})]^{\frac{1}{2}}$$

Constructive interference

$a = a_1 + a_2$, when $\theta_1 - \theta_2 = 0$, or in phase

$0, 2\pi, 4\pi, \ldots$

Destructive interference

$a = a_1 - a_2$ when $\theta_1 - \theta_2 = \pi$, or out of phase

$\pi, 3\pi, 5\pi, \ldots$