PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Review Sheet - Final Exam
May 11, 2010 (Tuesday) : 12 noon - 2 PM

A. Refer to Test-1 review sheet (the reduced version)

B. Refer to Test-2 review sheet (the reduced version)

C. Part-5 review sheet: Ch. 22, Ch. 23, Ch. 24.

For Final Exam
(Reduced Version)

A.

PHYS 306 Spring 2010
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Review Sheet - Test 1
Test 1: March 2, 2010 (Tuesday)

~~1. CH2: What is light? The historic and conceptual view of light.~~ ~~Snell's Law~~

2. CH7: Simple Harmonic Motion, Damped Harmonic Motion, Forced Vibration, Origin of Refractive Index

~~3. CH8: Fourier Series, Application of Fourier Series - a Plucked String, Introduction to Fourier Integral Theorem~~

~~4. CH9: Application of Fourier Integral Theorem~~

5. Ch11: Wave Functions, Energy Transport in Wave Motion, One-dimensional Wave Equation and its General Solution, Transverse Vibration of a Stretched String

6. CH10: Group Velocity, Dispersion, Group Velocity and Dispersion of a Wave Packet

~~7. CH12: Huygen's Principle~~

CH7:

$$x = a\cos(\omega t + \varphi)$$

$$m\frac{d^2x}{dt^2} = -kx \qquad \frac{d^2x}{dt^2} = -\frac{k}{m}x, \qquad \omega^2 = \frac{k}{m}$$

$$m\frac{d^2x}{dt^2} = F\cos\omega t - r\frac{dx}{dt} - k_0 x$$

$$x = a\cos(\omega t - \phi)$$

$$a = \frac{G}{\sqrt{(\omega_0^2-\omega^2)^2 + 4K^2\omega^2}} \qquad K = \frac{r}{m}, \quad G = \frac{F}{m}, \quad \omega_0 = \sqrt{\frac{k_0}{m}}$$

$$n = \frac{c}{v} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$$

$$n^2 = 1 + \frac{Nq^2}{m\varepsilon_0(\omega_0^2-\omega^2)}$$

~~CH8:~~ $f(t+nT) = f(t)$, T: period, $\omega = \frac{2\pi}{T}$

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

$$a_0 = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\,dt$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega t\,dt$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin n\omega t\,dt$$

~~CH9:~~ $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t')\,e^{i\omega(t-t')}\,dt'\,d\omega$$

$$F(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(t')\,e^{-i\omega t}\,dt$$

$$f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} F(\omega)\,e^{i\omega t}\,d\omega$$

CH 10: $y(x,t) = a\cos[kx \pm \omega t + \varphi]$

$$y(x,t) = a\cos[k(x \pm vt) + \varphi] \qquad v = \frac{\omega}{k}$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi(x,t)}{\partial t^2}$$

$$\psi(x,t) = f(x-vt) + g(x+vt$$

CH11: $v_p = \frac{\omega}{k}$, $v_g = \frac{d\omega}{dk}$, $\frac{1}{v_g} = \frac{1}{c}\left[n(\lambda_0) - \lambda_0\frac{dn}{d\lambda_0}\right]$

$$\Delta\tau_m = L\cdot\Delta\lambda_0\cdot D_m;\quad D_m = -\frac{1}{\lambda_0 c}\left[\lambda_0^2\frac{d^2n}{d\lambda_0^2}\right]$$

~~CH12:~~ $$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2-n_1}{R}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},\quad f = \frac{n_2-n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

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B

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Review Sheet - Test 2
Test 2: April 15, 2010 (Thursday)

CH13: Principle of Superposition; Stationary Wave on a String; Superposition of Two Sinusoidal Waves

Ch14: Interference Pattern of Water Surface Wave; Young's Double-Hole Experiment: Fringe Width; Light Intensity Distribution: Fringe Pattern; ~~Displacement of Fringes~~; Phase Change on Reflection

CH15: Interference by a Place Parallel Film; Non-reflecting Film; Newtown's Rings; Michelson Interferometer

~~CH16: Multiple Beam Interference from a Plane Parallel Film; Coefficient of Finesse; Transmittivity of Farby-Perot Etalon; FWHM; Fabry-Perot Interferometer; Spectrum Resolving Power~~

CH18: Single Slit Diffraction: Fringe Pattern; Diffraction by a Circular Aperture; Spatial Resolution; Two-slit Diffraction Pattern; N-Slit Diffraction Pattern: Principle Maxima; Diffraction Grating: Resolving Power

Test 2 - Review Apr. 13, 2010. (1)

CH13. Superposition

stationary wave $y = 2A\sin(kx)\cos(\omega t)$

nodal points: $y=0$ $kx = n\pi$, $x = n\cdot\frac{\lambda}{2}$

The loop length: $\frac{\lambda}{2}$

Two waves: the amplitude

$$a = [A_1^2 + A_2^2 + 2A_1A_2\cos(\theta_1-\theta_2)]^{\frac{1}{2}}$$

when $A_1 = A_2$, $a = 2A_1\cos^2\frac{\phi}{2}$, $\phi = \theta_1-\theta_2$

ϕ: phase difference between the two waves

CH14: Interference of two point sources

Optical path difference $\Delta = \overline{S_1P} - \overline{S_2P}$

phase difference: $\phi = \frac{2\pi}{\lambda}\Delta$

Young's Double Slit Experiment

$$\Delta = d\sin\theta = d\cdot\frac{y}{D} \quad ; \quad y = \frac{D}{d}\Delta$$

Constructive, maxima: $\Delta = n\lambda$,

Destructive, minima: $\Delta = (n+\frac{1}{2})\lambda$

Fringe width. $\beta = y_{n+1} - y_n = \frac{D}{d}\lambda$

Fringe Pattern: Intensity distribution of light

$$I = 4I_0\cos^2\frac{\delta}{2} \quad ; \quad \delta = \frac{2\pi}{\lambda}\Delta \text{ : the phase difference}$$

~~# Displacement of fringe~~

$$\cancel{\Delta = \overline{S_1P} - \overline{S_2P} + (n-1)t = -\frac{yd}{D} + (n-1)t}$$

$$\cancel{t = y_0\frac{d}{D(n-1)}} \; ; \; \cancel{y_0\text{: shift of fringe center}}$$

Abrupt phase change of π: light reflected by a denser medium

~~# Stoke's relation~~

$$\cancel{r_2 = -r_1}$$

$$\cancel{t_1t_2 = 1 - r_1^2 = 1 - r_2^2}$$

(2)

CH15 : Interference from Two Beams : spliting same source

Interference by plane parallel film.

Air - Film - Air

$\Delta = 2nd$

+ Abrupt phase change of π from the ~~2nd~~ beam

$\Rightarrow$ Constructive $\Delta = (m + \frac{1}{2})\lambda$

Destructive $\Delta = m\lambda$

~~# Oblique Incidence on a film~~

~~$\Delta = 2n_2 d \cos\theta'$~~

Non-reflecting film

Air — Film - Glass $n_a < n_f < n_g$.

phase change of π for both beam.

Destructive $\Delta = (m + \frac{1}{2})\lambda = \frac{1}{2}\lambda$

Film thickness $d = \frac{\Delta}{2n} = \frac{\lambda}{4n}$

Reflectivity $R = r^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

~~Tran~~ Newton's Ring.

Dark rings' position $r_m^2 = m\lambda R$

Assuming perfect contact at the center

Michelson Interferometer

$\Delta = 2d\cos\theta$: optical path difference

$\Delta = m\lambda$: Destructive. (π-phase change)

$\Delta = (m + \frac{1}{2})\lambda$: constructive.

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~~CH16: Multiple-Beam Interferometry~~

- ~~Transmittivity of the Fabry-Perot Interferometer~~

~~$T = \frac{1}{1+F\sin^2\frac{\delta}{2}}$~~

~~$F = \frac{4R}{(1-R)^2}$, the coefficient of Finesse~~

~~$R = r^2 = \left(\frac{n_1-n_2}{n_1+n_2}\right)^2$, reflectivity of the glass~~

~~$\delta = \frac{2\pi}{\lambda}\Delta$: phase difference between two beams~~

~~$\Delta = 2n_2 h\cos\theta_2$~~

~~Normal incidence, $\Delta = 2nh$, $\delta = \frac{4\pi nh}{\lambda}$~~

- ~~$\Delta\delta = \frac{4}{\sqrt{F}} = \frac{2(1-R)}{\sqrt{R}}$: FWHM of Interference peak.~~

- ~~Spectrum resolution, $\Delta\lambda \sim \Delta\delta$~~

~~$\Delta\lambda = \frac{\lambda^2}{4\pi nh}\Delta\delta = \frac{\lambda^2}{\pi nh\sqrt{F}}$~~

- ~~Resolving Power $P = \left|\frac{\lambda_0}{\Delta\lambda_0}\right| = \left|\frac{\nu_0}{\Delta\nu_0}\right| = \frac{\pi nh\sqrt{F}}{\lambda_0}$~~

CH17: Fraunhofer Diffraction

- Single slit: $I = I_0\frac{\sin^2\beta}{\beta^2}$

$$\beta = \frac{2\pi}{\lambda}\cdot\underbrace{\left(\frac{b}{2}\right)\sin\theta}_{\Delta} = \frac{\pi b\sin\theta}{\lambda}$$

First minima: $\theta_1 = \frac{\lambda}{b}$

- Circular Apeture: $\Delta\theta = 1.22\frac{\lambda}{D}$, also spatial Resolution
- Two-slit: $I = 4I_0\frac{\sin^2\beta}{\beta^2}\cos^2\gamma$

$$\gamma = \frac{2\pi}{\lambda}\underbrace{\left(\frac{d}{2}\right)\sin\theta}_{} = \frac{\pi d\sin\theta}{\lambda}, \quad \theta_1 = \frac{\lambda}{d}$$

- N-slit: $I = I_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$; $\theta_1 = \frac{\lambda}{Nd}$
- Diffraction grating: $d\sin\theta_m = m\lambda$; $R = \frac{\lambda}{\delta\lambda} = mN$

C: Part-5 Review Sheet

1. CH22: Polarization, Linear Polarization, Circular Polarization, Brewster's Angle, Double Refraction, Quarter Wave Plate (QWP)

2. CH23: Four Maxwell Equations, Constitutive Relation, Plane Wave in a Dielectric, Poynting Vector, Energy Density, Intensity, Continuity Conditions

3. CH24: Reflection and Refraction at the Interface of Two Dielectrics, Snell's Law, Brewster's Law, Amplitude Reflection Coefficient, "Intensity" Reflection Coefficient, π-phase change of reflection

Part-5 Review For Final Exam ①

CH 22.

Linear polarization (string)

$$\begin{cases} x(z,t) = a\cos(kz - \omega t + \phi_0) \\ y(z,t) = 0 \end{cases}$$

State of polarization of light

$$\begin{cases} E_x = a_1 \cos(kz - \omega t + \theta_1) \\ E_y = a_2 \cos(kz - \omega t + \theta_2) \end{cases}$$

$\theta = \theta_2 - \theta_1$

$\theta = 0$: Linear polarization (LP)

$\theta = \frac{\pi}{2}$: Right Circular polarization (RCP) $a_1 = a_2$

elliptical polarization $a_1 \neq a_2$

$\theta = \pi$: Linear polarization (LP)

$\theta = \frac{3}{2}\pi$: Left Circular polarization (LCP)

* Double Refraction

Ordinary ray : n_o ; $v_o = \frac{c}{n_o}$

Extraordinary ray : n_e ; $v_e = \frac{c}{n_e}$

* QWP : $\Delta\theta = \frac{\omega}{c}(n_o - n_e)d = \frac{\pi}{2}$

$$LP \xrightleftharpoons[QWP]{} CP$$

CH 23.

MX Equations

$$\begin{cases} ① \ \nabla \cdot \vec{D} = \rho \\ ② \ \nabla \cdot \vec{B} = 0 \\ ③ \ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ ④ \ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \end{cases}$$

Constitutive Relations

① $\vec{D} = \varepsilon \vec{E}$

② $\vec{B} = \mu \vec{H}$

③ $\vec{J} = \sigma \vec{E}$

Dielectric

$\varepsilon = \varepsilon$

$\mu = \mu_0$

$\sigma = 0 \Rightarrow \vec{J} = 0$

CH 23 (Contined)

$$\Rightarrow \nabla^2 \vec{E} = \varepsilon \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\varepsilon \mu_0}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

plane wave

$$\vec{K} \cdot \vec{E} = 0$$

$$\vec{K} \cdot \vec{H} = 0$$

$$\vec{H} = \frac{\vec{K} \times \vec{E}}{\omega \mu_0} \quad \text{or } H = \frac{k}{\omega \mu_0} E$$

$$\vec{E} = \frac{\vec{H} \times \vec{K}}{\omega \varepsilon} \quad \text{or } E = \frac{k}{\omega \varepsilon} H$$

$\Rightarrow \vec{E}, \vec{H}, \vec{K}$ orthogonal to each other

$\vec{S} = \vec{E} \times \vec{H}$; $\langle S \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} E_0^2$

$\vec{u} = \frac{1}{2}\vec{D} \cdot \vec{E} + \frac{1}{2}\vec{B} \cdot \vec{H}$, $\langle u \rangle = \frac{1}{2} \varepsilon E_0^2 = \frac{1}{2\mu_0} B_0^2$

$I = v \langle u \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} E_0^2$

$B_{1n} = B_{2n}$; $D_{1n} = D_{2n}$; $H_{1t} = H_{2t}$; $E_{1t} = E_{2t}$

CH 24

$$\omega_1 = \omega_2 = \omega_3 = \omega$$

$$k_{1z} = k_{2z} = k_{3z} \Rightarrow k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3$$

$$k_1 = k_3, \quad \theta_1 = \theta_3$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{k_2}{k_1} = \frac{\frac{1}{v_2}}{\frac{1}{v_1}} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

$$\begin{cases} r_{\parallel} = \dfrac{E_{30}}{E_{10}} = \dfrac{\varepsilon_2 \sin\theta_2 \cos\theta_1 - \varepsilon_1 \sin\theta_1 \cos\theta_2}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \\[2ex] t_{\parallel} = \dfrac{E_{20}}{E_{10}} = \dfrac{2\varepsilon_1 \sin\theta_1 \cos\theta_1}{\varepsilon_2 \sin\theta_2 \cos\theta_1 + \varepsilon_1 \sin\theta_1 \cos\theta_2} \\[2ex] r_{\perp} = -\dfrac{\sin(\theta_1 - \theta_2)}{\sin(\theta_1 + \theta_2)} \\[2ex] t_{\perp} = \dfrac{2 \sin\theta_2 \cos\theta_1}{\sin(\theta_1 + \theta_2)} \end{cases}$$

$$\boxed{\begin{array}{c} \theta_p ; \ \tan\theta_p = \dfrac{n_2}{n_1} \\ r_{\parallel} = 0 \\ r_{\perp} \neq 0 \end{array}}$$