PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Test 2

April 15, 2010

There are five questions. You need to answer four of them. You must indicate which one of the five you omit.

1. (25pts) Standing waves are formed on a stretched string under tension of 10^6 dyn. The mass per unit length of the string is 0.1 g/cm. The length of the string is 50 cm. If it vibrates with five loops,
(1) What is the wavelength of the standing wave?
(2) What is the wave velocity along the string?
(3) What is the angular frequency of the vibration?

Answer:
(1) For the standing wave in a stretched string, the length of the loop is half of the wavelength.

$$L_{loop} = \frac{length\_of\_string}{number\_of\_loops} = \frac{50}{5} = 10\,\text{cm}, \quad \text{therefore}$$
$$\lambda = 20\,\text{cm}$$

(2) The wave velocity along a stretched string is determined by the tension force and string mass density, which is described as

$$\mathrm{V} = \sqrt{\frac{\mathrm{T}}{\rho}} = \sqrt{\frac{10^6}{0.1}} = 3.16\times 10^3 \;\; \text{cm/s} \;\; \text{(note : all quantities are in CGS unit)}$$

(3)

$$\omega = 2\pi\upsilon = 2\pi\cdot(\frac{V}{\lambda}) = 2\times 3.14\times\frac{3.16\times 10^3}{20} = 992.2 \quad \text{rad/sec}$$

2. (25 pts) In Young's double-hole experiment, the distance between the two holes is 1.0 mm, λ=5 ×10^{-5} cm, and D = 100 cm.
(1) What will be the fringe width?
(2) Calculate I/I_{max} where I represents the intensity at a point where the path difference is λ/5?

Answer:
(1) For Young's double hole experiment, the path difference is

$$\Delta = d \sin\theta = d \frac{y}{D}$$

The locations of bright fringes, due to constructive interference, are related with

$$\Delta = m\lambda = d \frac{y_m}{D}; \text{ Therefore,}$$

$$y_m = m\lambda \frac{D}{d}$$

The fringe width is

$$\Delta y = \lambda \frac{D}{d} = 5 \times 10^{-5} \times \frac{100}{0.1} = 0.05 \text{ cm}$$

(2) The fringe pattern for Young's double-hole experiment in terms of the phase difference δ is

$$I = 4I_0 \cos^2(\frac{\delta}{2}) = I_{\max} \cos^2(\frac{\delta}{2})$$

The phase difference δ is related with the path difference Δ as

$$\delta = 2\pi \frac{\Delta}{\lambda} = \frac{2\pi}{5}$$

$$\frac{I}{I_{\max}} = \cos^2(\frac{\delta}{2}) = \cos^2(\frac{\pi}{5})$$

$$\frac{I}{I_{\max}} = 0.65$$

3. (25 pts) (1) Consider a nonreflecting film of refractive index 1.60. Assuming that its thickness is 8×10^{-6} cm. Calculate the wavelength (in the visible region) for which the film will be nonreflecting?

(2) In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.06 mm, 200 fringes cross the field of view. Calculate the wavelength?

Answer:

(1) The non-reflecting film is achieved through destructive interference in the film. The condition is satisfied, when

$$\Delta = 2nd = \frac{\lambda}{2}$$

$$\lambda = 4nd = 4\times1.60\times8\times10^{-6} = 5.12\times10^{-5}\ \text{cm}$$

$$\lambda = 512\ \text{nm}$$

(2) In the case of Michelson Interferometer, the path difference is

$$\Delta = 2d$$

Considering the dark fringes (without losing generality)

$$\Delta = m\lambda.\ \text{Therefore,}$$

$$\delta\Delta = \delta m\cdot\lambda.$$

We know that

$$\delta\Delta = 2\delta\text{d} = 2\times0.06 = 0.12\ \text{mm} = 0.012\ \text{cm}$$

$$\delta\text{m} = 200$$

$$\lambda = \frac{\delta\Delta}{\delta\text{m}} = \frac{0.012}{200} = 6.0\times10^{-5}\ \text{cm}$$

$$\lambda = 600\ \text{nm}$$

4. (25 pts) For a scanning Fabry-Perot Interferometer, the transmitted interference pattern is described as, for a normal incidence,

$$T = \frac{1}{1+F\sin^2\frac{\delta}{2}}; \quad \delta = \frac{4\pi h}{\lambda_0}$$

(1) For what δ_m, the transmittivity T equals 1?
(2) Show that

$$\Delta\delta = \frac{4}{\sqrt{F}}$$

in which, $\Delta\delta$ represents the FWHM of the transmittivity; in other words, T=1/2 at $\delta = \delta_m + \Delta\delta/2$

(3) Show that the spectrum resolution

$$\left|\Delta\lambda_0\right| = \frac{\lambda_0^{\ 2}}{\pi h\sqrt{F}}$$

Answer:

(1) For T=1

$$\sin\frac{\delta}{2} \text{ must equal zero. In other words,}$$

$$\frac{\delta_m}{2} = m\pi, \quad m = 0,1,2...$$

$$\delta_m = 2m\pi, \quad m = 0,1,2...$$

(2)

$$T = 1/2 \text{ means that}$$

$$F\sin^2\frac{\delta}{2} = 1$$

$$\sin^2\frac{\delta}{2} = \frac{1}{F}; \quad \sin\frac{\delta}{2} = \frac{1}{\sqrt{F}}$$

$$\text{Since } \delta = \delta m + \frac{\Delta\delta}{2} = 2m\pi + \frac{\Delta\delta}{2}$$

$$\sin\frac{\delta}{2} = \sin(m\pi + \frac{\Delta\delta}{4})$$

Using the identity: $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\sin(m\pi + \frac{\Delta\delta}{4}) = \sin(m\pi)\cos(\frac{\Delta\delta}{4}) + \cos(m\pi)\sin(\frac{\Delta\delta}{4}) = \sin(\frac{\Delta\delta}{4})$$

Further, for small Δδ: $\sin(\frac{\Delta\delta}{4}) = \frac{\Delta\delta}{4}$; therefore

$$\frac{\Delta\delta}{4} = \frac{1}{\sqrt{F}}; \text{Therefore,}$$

$$\Delta\delta = \frac{4}{\sqrt{F}}$$

(3)

For a FB interferometer, we know that

$$\delta = \frac{4\pi h}{\lambda_0}$$

The phase difference δ is caused either by the change of h, or by the change of wavelength. For studying the spetrum resolution, we assume that h is fixed, and allow λ_0 varies:

$$\Delta\delta = \Delta(\frac{4\pi h}{\lambda_0}) = -\frac{4\pi h}{{\lambda_0}^2}\Delta\lambda_0$$

$$\Delta\lambda_0 = -\frac{{\lambda_0}^2}{4\pi h}\Delta\delta = -\frac{{\lambda_0}^2}{\pi h\sqrt{F}}; \text{ Therefore,}$$

$$|\Delta\lambda_0| = \frac{{\lambda_0}^2}{\pi h\sqrt{F}}$$

5. (25 pts) A convex lens of focal length 50 cm is placed after a slit of width 0.6 mm. If a plane wave of wavelength 6000 Å falls normally on the slit, calculate the separation between the first minima on either side of the central maximum.

Answer:

This is the case of single-slit diffraction. The diffraction pattern is

$$I = I_0 \frac{\sin^2 \beta}{\beta^2};$$

$$\beta = \frac{2\pi}{\lambda}\frac{b}{2}\sin\theta = \frac{\pi b \sin\theta}{\lambda}$$

The central maximum occurs at $\theta=0$, and $\beta=0$

The minima occurs at

$$\sin\beta = 0$$

$$\beta = m\pi,\ \mathrm{m} \neq 0, \mathrm{m} = \pm 1, \pm 2....$$

For the first minimum,

$$\beta = \pi = \frac{\pi b \sin\theta}{\lambda}$$

$$\sin\theta = \frac{\lambda}{b} = \frac{6.0\times 10^{-5}(cm)}{0.06(cm)} = 10^{-3}\ \text{rad, or}$$

$$\theta = 10^{-3}\ \text{rad}$$

The distance between the two minima on either side is

$$\Delta \mathrm{y} = 2\times \mathrm{f} \times \theta = 2\times 50\times 10^{-3} = 0.1\,\mathrm{cm}$$