PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Test 1

March 2, 2010

Answer each of the following four questions.

1. Consider the simple harmonic motion of a pendulum. The bob of the pendulum of mass M is attached to a string of length L.
(1) Show the equation of motion of the pendulum.
(2) Show that the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

(3) If the string is one meter long, what is the period? and what is the angular frequency?

2. Consider the periodic sawtooth function of the form

$f(t) = t$ for $-\tau < t < \tau$

$f(t + 2n\tau) = f(t)$

with the period $T = 2\tau$,

and the *fundamental-frequency* $\omega = \pi/\tau$

Show that the above function can be expanded in a Fourier series of the form

$$f(t) = \frac{2\tau}{\pi}(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - .....)$$

3. The displacement function associated with a monochromatic wave is given by

$$y(x,t) = 3.0\cos(4.0x - 2.0t)$$

where x and y are measured in meters and t in seconds.
(1) What is the spatial frequency k? and what is the angular frequency ω?
(2) Calculate the wavelength λ, and period T?
(3) Calculate the propagation velocity of the wave? Which direction does the wave move?

4. An ionized gas or plasma is a dispersive medium for EM waves. Given the dispersion relation

$$\omega^2 = \omega_p{}^2 + c^2k^2$$

where ω_p is a constant, the "plasma frequency", and c is the speed of light in vacuum.
Determine an expression for the group velocity in this medium in term of the variable ω?