PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment

HW#11
Assignment Date: Apr. 27, 2010
Due Date: May 4, 2010

1. Discuss the state of polarization when the x and y components of the electric field are given by the following equations. In each case, plot the rotation of the tip of the electric vector on the plane $z = 0$. (Q22.1):

(a)
$$\left.\begin{aligned} E_x &= E_0 \cos(\omega t + kz) \\ E_y &= \frac{1}{\sqrt{2}} E_0 \cos(\omega t + kz + \pi) \end{aligned}\right\}$$

(b)
$$\left.\begin{aligned} E_x &= E_0 \sin(\omega t + kz) \\ E_y &= E_0 \cos(\omega t + kz) \end{aligned}\right\}$$

(c)
$$\left.\begin{aligned} E_x &= E_0 \sin\left(kz - \omega t + \frac{\pi}{3}\right) \\ E_y &= E_0 \sin\left(kz - \omega t - \frac{\pi}{6}\right) \end{aligned}\right\}$$

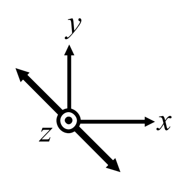
(d)
$$\left.\begin{aligned} E_x &= E_0 \sin\left(kz - \omega t + \frac{\pi}{4}\right) \\ E_y &= \frac{1}{\sqrt{2}} E_0 \sin(kz - \omega t) \end{aligned}\right\}$$

Answer:

1 (a) Propagation along the $-z$ direction (into the page). At $z = 0$

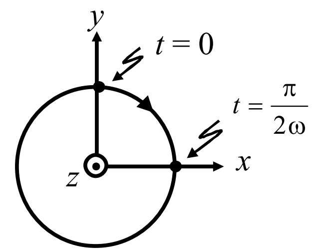
$$E_x = E_0 \cos\omega t; \; E_y = -\frac{1}{\sqrt{2}} E_0 \cos\omega t$$

$$\frac{E_x}{E_y} = -\sqrt{2} = \tan(125.3^\circ)$$



LP (Linear Polarized) along the direction shown in the figure; the propagation is along the $-z$ axis; i.e., into the page.

(b) Propagation along the $-z$ direction (into the page). At $z = 0$

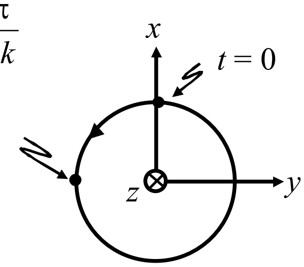


$$E_x = E_0 \sin \omega t; \quad E_y = E_0 \cos \omega t$$

$$\Rightarrow E_x^2 + E_y^2 = E_0^2 \Rightarrow \text{Circularly polarized}$$

Since propagation is along the $-z$ axis; i.e., into the page, we have a RCP wave.

(c) Propagation is along the $+z$ direction (into the page). At $z = \dfrac{\pi}{6k}$



$$E_x = E_0 \sin\left(\frac{\pi}{6} - \omega t + \frac{\pi}{3}\right) = E_0 \cos \omega t$$

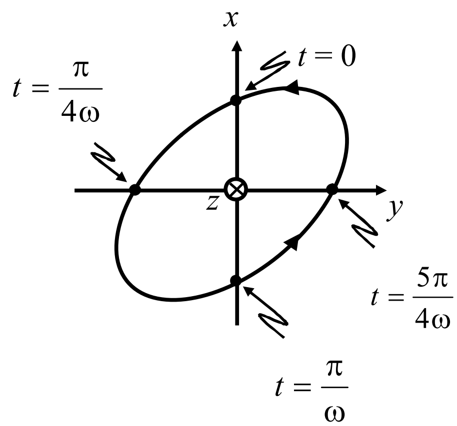
$$E_y = -E_0 \sin \omega t$$

$$\Rightarrow E_x^2 + E_y^2 = E_0^2 \Rightarrow \text{Circularly polarized wave}$$

Since propagation is along the $+z$ axis; i.e., into the page, we have a LCP wave.

(d) Propagation is along the $+z$ axis (into the page). Now, at $z = 0$

$$E_x = E_0 \sin\left(\frac{\pi}{4} - \omega t\right); \qquad E_y = -\frac{E_0}{\sqrt{2}} \sin \omega t$$



Thus at $t = 0$; $\quad E_x = \dfrac{E_0}{\sqrt{2}}$ and $E_y = 0$

$t = \dfrac{\pi}{4\omega}$; $\quad E_x = 0$ and $E_y = -\dfrac{E_0}{2}$

$t = \dfrac{\pi}{2\omega}$; $\quad E_x = -\dfrac{E_0}{\sqrt{2}}$ and $E_y = -\dfrac{E_0}{\sqrt{2}}$

$t = \dfrac{3\pi}{4\omega}$; $\quad E_x = -E_0$ and $E_y = -\dfrac{E_0}{2}$

$t = \dfrac{\pi}{\omega}$; $\quad E_x = -\dfrac{E_0}{\sqrt{2}}$ and $E_y = 0$

$t = \dfrac{5\pi}{4\omega}$; $\quad E_x = 0$ and $E_y = +\dfrac{E_0}{2}$

The wave is LEP (Left Elliptically Polarized) as shown in the figure. Further

$$E_x = \frac{E_0}{\sqrt{2}} \cos \omega t - \frac{E_0}{\sqrt{2}} \sin \omega t$$

or

$$E_x - E_y = \frac{E_0}{\sqrt{2}} \cos \omega t \quad \text{and} \qquad E_y = -\frac{E_0}{\sqrt{2}} \sin \omega t$$

Thus $E_x^2 + 2E_y^2 - 2E_x E_y = \dfrac{E_0^2}{2}$ which represents ellipse in the $E_x - E_y$ plane.

2. For calcite, the values of n_o and n_e for λ_0 = 4046Å are 1.68134 and 1.49694 respectively; corresponding to λ_0 = 7065Å the values are 1.65207 and 1.48359 respectively. We have a calcite quarter-wave plate corresponding to λ_0 = 4046Å. A left-circularly polarized beam of λ_0 = 7065Å is incident on this plate. Obtain the state of polarization of the emergent beam (Q22.13).

Answer:

For λ_0 = 4046 Å, the thickness of the QWP is given by [see Eq. (50)]

$$d = \frac{\lambda_0}{4(n_0 - n_e)} = \frac{4.046 \times 10^{-5}}{4(1.68134 - 1.49694)} \approx 5.49 \times 10^{-5} \text{ cm}$$

At λ_0 = 7065 Å, the phase difference introduced is given by:

$$\theta = \frac{2\pi}{\lambda_0}(n_0 - n_e)d = \frac{2\pi}{7065 \times 10^{-5}} \times \frac{4.046 \times 10^{-5}}{4} \approx \frac{\pi}{3.49} \approx \frac{\pi}{4} \text{ (say)}$$

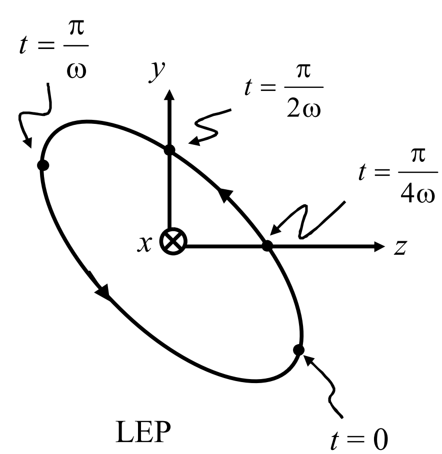
The LCP is incident on the QWP is given by

$$E_y = E_0 \sin \omega t; \; E_z = E_0 \cos \omega t$$

The output beam will be

$$E_y = E_0 \sin\left(\omega t - \frac{\pi}{4}\right); \; E_z = E_0 \cos \omega t$$

which is a LEP beam.



3. Starting from the four Maxwell equations in a homogeneous dielectric medium (Eq, 11, 12, 13 and 14 in Chap. 23), for a given plane wave propagating in the +z direction,

$$\vec{E} = \hat{x}E_0 \cos(kz - wt)$$
$$\vec{H} = \hat{y}H_0 \cos(kz - wt)$$

Show that

$$\frac{\omega}{k} = \frac{1}{\sqrt{\varepsilon\mu_0}}$$

(Note that this exercise is to derive the wave velocity from Maxwell equations, since $V = \frac{\omega}{k}$)

Answer:

Refer to the textbook, or the class notes.

4. On the surface of the earth we receive about 1.37 kW of energy per square meter from the sun. Calculate the electric field associated with the sunlight (on the surface of the earth) assuming that it is essentially monochromatic with λ= 6000Å . (Q23.1)

Answer:

$$I \approx 1.37\times10^{3}\ \text{W/m}^{2} = \frac{1}{2}\varepsilon_0 c E_0^2$$

$\Rightarrow$

$$E_0 \approx \sqrt{\frac{2\times1.37\times10^{3}}{8.854\times10^{-12}\times3\times10^{8}}} \approx 1016\ \text{V/m}$$