PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment - Solution

HW#10
Assignment Date: Apr. 13, 2010
Due Date: Apr. 20, 2010

1.(30pts) A circular aperture of radius 0.01 cm is placed in front of a convex lens of focal length of 25 cm and illuminated by a parallel beam of light of wavelength 5×10^{-5} cm. Calculate the radius of the first dark ring (Q18.5).

Answer:

Using the diffraction formula of circular aperture, for dark rings:

$$\sin\theta = \frac{\nu\lambda}{2\pi a}, \text{ where } \nu = 3.832, 7.016....$$

For the firs dark ring

$$\sin\theta = \frac{3.832\times\lambda}{2\pi a} = 0.61\frac{\lambda}{a} = 1.22\frac{\lambda}{D}$$

where a the radius, and D the diameter of the circular aperture. Therefore,

$$\sin\theta = 0.61\frac{5\times10^{-5}}{0.01} = 3.05\times10^{-3} \text{ rad}$$

Thus, the radius of the first dark ring

$$\text{r} = \text{f } \tan\theta \approx \text{f } \sin\theta = 25\times3.05\times10^{-3} = 0.076\,\text{cm}$$

2. (40 pts) Consider a set of two slits each of width $b = 5 \times 10^{-2}$ cm and separated by a distance d = 0.1 cm, illuminated by a monochromatic light of wavelength 6.328×10^{-5} cm. If a convex lens of focal length 10 cm is placed beyond the double slit arrangement, calculate (adapted from Q18.7)

(1) the first diffraction minimum

(2) the first interference minimum inside the first diffraction fringe

(3) the first interference maximum inside the first diffraction fringe.

Answer:

(1) The double slit diffraction pattern is described by

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma,$$

which is the product of the one-slit diffraction and the double-hole interference pattern

$$\beta = \frac{\pi b \sin\theta}{\lambda} \quad \text{; the (half) phase of diffraction term}$$

$$\gamma = \frac{\pi d \sin\theta}{\lambda} \quad \text{; the (half) phase of interference term}$$

The first diffraction minimum occurs at

$$\sin \beta = 0, \quad \text{and } \beta = \pi$$

$$\sin\theta = \frac{\lambda}{\text{b}} = \frac{6.328 \times 10^{-5}}{5 \times 10^{-2}} = 1.27 \times 10^{-3} \text{ rad}$$

$$\text{x} = \text{f } \tan\theta \approx \text{f } \sin\theta = 10 \times 1.27 \times 10^{-3} = 0.013 \text{ cm}$$

(2) The first interference minimum

$$\cos \gamma = 0, \quad \text{and } \gamma = \frac{\pi}{2}$$

$$\sin\theta = \frac{\lambda}{2\text{d}} = \frac{6.328 \times 10^{-5}}{2 \times 0.1} = 3.16 \times 10^{-4} \text{ rad}$$

$$\text{x} = \text{f } \tan\theta \approx \text{f } \sin\theta = 10 \times 3.16 \times 10^{-4} = 0.0032 \text{ cm}$$

(3) The first interference maximum

$$\cos \gamma = 1, \quad \text{and } \gamma = \pi$$

$$\sin\theta = \frac{\lambda}{\text{d}} = \frac{6.328 \times 10^{-5}}{0.1} = 6.33 \times 10^{-4} \text{ rad}$$

$$\text{x} = \text{f } \tan\theta \approx \text{f } \sin\theta = 10 \times 6.33 \times 10^{-4} = 0.0063 \text{ cm}$$

3. (30 pts) Calculate the resolving power in the second order spectrum of a 1 inch grating having 15,000 lines. (Q18.14)

Answer:

The resolving power of the grating device is described as

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

$$R = 2 \times 15000 = 30000$$