PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment - Solution

HW#9
Assignment Date: Apr. 06, 2010
Due Date: Apr. 13, 2010

1. (20 pts) Calculate the minimum spacing between the plates of a Fabry-Perot interferometer which would resolve two lines with $\Delta\lambda = 0.1$ Å at $\lambda = 6000$ Å. Assume the reflectivity to be 0.8. (Q16.2 in CH16)

Answer:

Using the resolving power formula of Fabry-Perot Interferometer

$$P = \frac{\lambda_0}{\Delta\lambda} = \frac{\pi h\sqrt{F}}{\lambda_0}$$

The larger the spacing h, the higher the resolution

$$P = \frac{6000}{0.1} = 6.0\times10^4$$

$$F = \frac{4R}{(1-R)^2} = \frac{4\times0.8}{(1-0.8)^2} = 80$$

$$h = \frac{P\lambda_0}{\pi\sqrt{F}} = \frac{6.0\times10^4\times6.0\times10^{-5}(cm)}{3.142\times\sqrt{80}}$$

Therefore,

$$\mathrm{h}_{\min} = 0.128\,\mathrm{cm}$$

2. (20 pts) Consider a monochromatic beam of wavelength 6000 Å incident normally on a scanning Fabry-Perot interferometer with n_2 = 1 and F = 400. The distance between the two mirrors is written as $h = h_0 + x$. With h_0 = 10 cm, calculate

(a) The first three values of x for which we will have unit transmittivity and the corresponding value of m.

(b) Also calculate the FWHM Δh for which the transmittivity will be half. (Q16.5 in CH16)

Answer:

(a) $T = \dfrac{1}{1 + F\sin^2\frac{\delta}{2}}; \quad \delta = \dfrac{4\pi}{\lambda_0} n_2 h \cos\theta_2$

$\lambda_0 = 6 \times 10^{-5}$ **cm,** $n_2 = 1$**,** $h = h_0 + x$ **with** $h_0 = 10$ **cm**

and for normal incidence $\theta_2 = 0$**.**

For $T = 1$**, or complete transmission, we must have**

$$\delta = 2m\pi, \quad \text{m is an integer}$$

$$\delta = \frac{4\pi}{6\times10^{-5}} \times 10 \times \left(1 + \frac{x}{h_0}\right) = 2m\pi$$

$$\Rightarrow m = \frac{10^6}{3}\left(1 + \frac{x}{h_0}\right) = 333333.333\left(1 + \frac{x}{h_0}\right)$$

Thus for $x = 0$**,** m **is not an integer and** $T \neq 1$**.**

The first maximum will occur when

$$m = 333334 = \frac{10^6}{3}\left(1 + \frac{x}{h_0}\right), \quad \textbf{or}$$

$$2 = 10^6 \frac{x}{h}$$

$$\Rightarrow x = 2 \times 10 \times 10^{-6} \text{ cm} = 200\,\text{nm}$$

Similarly, the second and third maxima will occur when

$$m = 333335 = \frac{10^6}{3}\left(1 + \frac{x}{h_0}\right) \Rightarrow x = 500 \text{ nm}$$

and

$$m = 333336 = \frac{10^6}{3}\left(1 + \frac{x}{h_0}\right) \Rightarrow x = 800 \text{ nm}$$

respectively.

(b) The FWHM in terms of phase difference is given by [see Eq. (8) in CH16]

$$\Delta\delta = \frac{4}{\sqrt{F}}$$

For normal incidence and n_2 =1,

$$\delta = \frac{4\pi}{\lambda_0} h \textbf{.}$$

Thus

$$\Delta\delta = \frac{4\pi}{\lambda_0}\Delta h = \frac{4}{\sqrt{F}}$$

$$\Rightarrow \Delta h = \frac{\lambda_0}{\pi\sqrt{F}} = \frac{6\times10^{-5}\,(cm)}{3.142\times\sqrt{400}} = 9.55\times10^{-7}\text{ cm}$$

$$\Delta h = 9.55\text{ nm}$$

3. (20 pts) In continuation of Problem 2, consider now two wavelengths λ_0 (= 6000 Å) and $\lambda_0 + \Delta\lambda$ incident normally on the Fabry-Perot interferometer with $n_2 = 1$, $F = 400$ and $h_0 = 10$ cm. What will be the value of $\Delta\lambda$ so that $T = ½$ occurs at the same value of h for both the wavelengths. (Q16.6 in CH16)

Answer:

Similarly, we start with the FWHM formula in terms of phase difference [see Eq. (8) in CH16]

$$\Delta\delta = \frac{4}{\sqrt{F}}$$

For normal incidence and n_2 =1,

$$\delta = \frac{4\pi}{\lambda_0} h$$

Apparently, there are two ways to change the phase, one by h, the other by the wavelength. If we fix the h

$$\Delta\delta = -\frac{4\pi h}{\lambda_0^{2}}\Delta\lambda_0, \quad \text{or}$$

$$\Delta\lambda_0 = -\frac{\lambda_0^{2}}{\pi h\sqrt{F}}$$

This is actually the formula for the spectral resolution of F-B interferometer

$$\Delta\lambda_0 = -\frac{(6\times10^{-5}\,cm)^2}{3.142\times10cm\times\sqrt{400}} = 5.7\times10^{-12}\ \text{cm}$$

Therefore, the spectrum resolution $\Delta\lambda$ is as small as $5.7\text{x}10^{-4}$ Å, which is an extremely small number. It proves that a F-B is an excellent spectrometer.

4. (20 pts) A plane wave (λ = 5000 Å) falls normally on a long narrow slit of width 0.5 mm. Calculate the angles of diffraction corresponding to the first three minima. (Q18.1 in CH18)

Answer:

The single slit diffraction pattern is

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}; \quad \beta = \frac{\pi b \sin \theta}{\lambda}$$

For the occurrence of minima

$$\beta = m\pi; \ \ \mathrm{m} = 1,2,3 \, (\mathrm{note\ m} \neq 0)$$

$$\sin \theta = \mathrm{m} \frac{\lambda}{\mathrm{b}}$$

Thus, for b = 0.5 mm=0.05 cm,

$$\theta_1 = \sin^{-1}\left(1 \times \frac{5 \times 10^{-5}}{0.05} \right) = 0..001 \ \ \mathrm{rad} = 0.057^\circ$$

$$\theta_2 = \sin^{-1}\left(2 \times \frac{5 \times 10^{-5}}{0.05} \right) = 0.002 \ \mathrm{rad} = 0.115^\circ$$

$$\theta_3 = \sin^{-1}\left(3 \times \frac{15 \times 10^{-5}}{0.05} \right) = 0.003 \ \ \mathrm{rad} = 0.17^\circ$$

5. (20 pts) A convex lens of focal length 20 cm is placed after a slit of width 0.6 mm. If a plane wave of wavelength 6000 Å falls normally on the slit, calculate the separation between the second minima on either side of the central maximum. (Q18.2 in CH18)

Answer:

Similarly as in question 4, the single slit diffraction pattern is

$$I = I_0 \frac{\sin^2 \beta}{\beta^2}; \quad \beta = \frac{\pi b \sin\theta}{\lambda}$$

For the occurrence of minima

$$\beta = m\pi; \ \mathrm{m} = \pm 1, \pm 2, \pm 3 \ (\mathrm{note\ m} \neq 0)$$

$$\sin\theta = \mathrm{m}\frac{\lambda}{\mathrm{b}}$$

From previous question, the second minima (m=-2 and m=2)

$$\theta_2 = \sin^{-1}\left(2 \times \frac{\lambda}{b}\right) = \sin^{-1}\left(2 \times \frac{6 \times 10^{-5} cm}{0.06 cm}\right) = 0.002 \text{ rad}$$

The angular distance is

$$\Delta\theta = 2 \times \theta 2 = 0.004 \text{ rad}$$

The separation distance Y is

$$Y = f \times \Delta\theta = 20(cm) \times 0.004 = 0.08 \text{ cm}$$