PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment - Solution

HW#8
Assignment Date: Mar. 30, 2010
Due Date: Apr. 06, 2010

1. (20pts) In the Newton rings arrangement, the radius of curvature of the curved side of the planeconvex lens is 100 cm. For $\lambda = 6\times10^{-5}$ cm, what will be the radii of the 9th and 10th bright rings? (Q15.4 in CH15)

Answer:

For Newton's ring, the m^{th} dark ring is related as

$$r_m{}^2 = m\lambda R, \quad \mathrm{m} = 0,1,2.....$$

While the m^{th} bright ring is related as

$$r_m{}^2 = (m - \frac{1}{2})\lambda R, \quad \mathrm{m} = 1,2.....$$

With this formula, it is obvious that the first ring occurs as m = 1
(when the thickness of the air film t = $\lambda/4$)

$$r_m = \sqrt{(m - \frac{1}{2})\lambda R}$$

$$r_9 = \sqrt{(9 - \frac{1}{2})(6\times10^{-5})100} = 0.2258\,\mathrm{cm}$$

$$r_{10} = \sqrt{(10 - \frac{1}{2})(6\times10^{-5})100} = 0.2387\,\mathrm{cm}$$

2. (20pts) In the Newton's rings arrangement, the radius of curvature of the curved surface is 50 cm. The radii of the 9th and 16th dark rings are 0.18 cm and 0.2235 cm. Calculate the wavelength. [Hint: The use of Eq. (66) will give wrong results, why?) (Q15.5 in CH15)

Answer:

Equation (66) for dark rings

$$r_m{}^2 = m\lambda R, \quad \mathrm{m} = 0,1,2.....$$

However, this equation is only valid for perfect contact between convex lens and the plane class, or the air film thickness at the center is zero.
In practice, this perfect contact is hard to achieve, which makes Eq 66 invalid. One would obtain two different wavelengths for the two orders if Eq 66 is used.

The more general equation for finding wavelength from Newton's ring pattern is Eq. 68:

$$\lambda = \frac{D^2{}_{m+p} - D_m{}^2}{4pR}$$

$$p = 16 - 9 = 7$$

$$R = 50cm$$

$$D_9 = 2 \times 0.18 = 0.36\,\mathrm{cm}$$

$$D_{16} = 2 \times 0.2235 = 0.447\,\mathrm{cm}$$

$$\lambda = \frac{0.447^2 - 0.36^2}{4 \times 7 \times 50} = 5.0149 \times 10^{-5}\,\mathrm{cm} = 5014.9\,\mathrm{A}$$

3. (20pts) In the Michelson interferometer arrangement, if one of the mirrors is moved by a distance 0.08 mm, 250 fringes cross the field of view. Calculate the wavelength? (Q15.9 in CH15)

Answer:

The optical path difference for the Michelson interferometer is

$$\Delta = 2d\cos\theta$$

For this problem, without losing generality, we could assume dark fringes at the center (θ=0), which yield

$$2d = m\lambda$$

$$\delta m = 250$$

$$\delta d = 0.08\ \text{mm}$$

$$\lambda = \frac{2\delta d}{\delta m} = \frac{2\times 0.08}{250} = 6.4\times 10^{-4}\ \text{mm}$$

$$\lambda = 6400\ \text{A}$$

4. (20pts) In the Michelson interferometer experiment, assuming $\lambda = 5\times10^{-5}$ cm and d = 5×10^{-3} cm, find the values of angle θ and fringe order m for the first, second and third bright rings from the center?

Answer:

The fringe pattern formula for bright rings is

$$2d\cos\theta = (m+\frac{1}{2})\lambda$$

$$\cos\theta = \frac{(m+\frac{1}{2})\times5\times10^{-5}}{2\times5\times10^{-3}}$$

$$\cos\theta = \frac{(m+\frac{1}{2})}{200}$$

$$\text{Since}\cos\theta <= 1,$$

$$\text{The first bright ring from the center is at m} = 199$$

$$\cos\theta = 0.9975$$

$$\theta = 0.0707\text{ rad} = 4.05°$$

$$\text{The second brigh ring is at m} = 198$$

$$\cos\theta = 0.9925$$

$$\theta = 0.1226\text{ rad} = 7.02°$$

$$\text{The third brigh ring is at m} = 197$$

$$\cos\theta = 0.9875$$

$$\theta = 0.1582\text{ rad} = 9.07°$$

5. (20pts) Show that, for multiple reflections in a plane parallel film, the transmittivity T of the film is given by

$$T = \frac{1}{1 + F\sin^2\dfrac{\delta}{2}}$$

Hint: refer to Ch16.2 or the class note.

Answer:

See the textbook Ch16.2, or the class note Lect. 16