PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment - Solution

HW#7
Assignment Date: Mar. 23, 2010
Due Date: Mar. 30, 2010

1. (25 pts) In Young's double-hole experiment, a thin mica sheet (n=1.5) is introduced in the path of the one of the beams. If the central fringe gets shifted by 0.5 cm, calculate the thickness of the mica sheet. Assume d = 0.1 cm and D = 100 cm.

Answer:

If a thin-sheet is introduced in S_1P, the optical path difference is

$$\Delta = S_1P - S_2P + (n-1)t = -y\frac{d}{D} + (n-1)t$$

For the central fringe, $\Delta = 0$, thus

$$t = y\frac{d}{D}\frac{1}{(n-1)} = 0.5\frac{0.1}{100}\cdot\frac{1}{(1.5-1)} = 1.0\times10^{-3}\text{ cm}$$

2. (25 pts) In Young's double-hole experiment, interference fringes are formed using sodium light which predominantly comprises two wavelengths (5890 and 5896 Å). Find the location on the screen (or the distance from the fringe center) where the fringe pattern will disappear. Assume d=0.05 cm and D=100 cm.

Answer:

The fringe pattern disappears at the location where, for certain order of fringe m, is the maximum intensity for one wavelength, while the minimum intensity for the other wavelength.

Let $\lambda_1 = 5.896\times10^{-5}$ cm and $\lambda_2 = 5.890\times10^{-5}$ cm. Thus $\lambda_1 > \lambda_2$. We must have

$$\Delta = S_2P - S_1P = m\lambda_1 = \left(m + \frac{1}{2}\right)\lambda_2$$

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$$m = \frac{\lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{5.890\times10^{-5}}{2\times(0.006\times10^{-5})} = 490.8$$

$$\Delta = m\lambda_1 = 0.0289\,\text{cm}$$

If the location y << D, we have

$$\Delta = y\frac{d}{D}$$

$$y = \Delta\frac{D}{d} = 0.0289\cdot\frac{100}{0.05} = 57.8\,\text{cm}$$

It is acceptable if you have reached the answer of 57.8 cm.

However, a better answer is to consider that the assumption of y << D is not valid. We must use the accurate expression [CH14, Eq. (23)] to calculate *y*.

$$y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\left[D^2 + \frac{1}{4}\left(d^2 - \Delta^2\right)\right]^{1/2} \approx \pm 72\ \text{cm}$$

3. (25 pts) In Young's double-hole experiment, calculate I/I_{max} where I represents the intensity at a point where the path difference is
(1) 0
(2) λ/5
(3) λ/2
(4) 3λ

Answer:
For the interference of two light waves, the general expression for intensity fringe pattern is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where δ is the phase difference. In Young's experiment, we can assume

$I_1 = I_2 = I_0$, thus

$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\dfrac{\delta}{2}$

and $I_{max} = 4I_0$

$I/I_{max} = \cos^2\dfrac{\delta}{2}$

The relation between path difference Δ and phase difference δ is

$\delta = 2\pi\dfrac{\Delta}{\lambda}$

(1) $\Delta = 0,\quad \delta = 0,\quad I/I_{max} = \cos^2(0) = 1;$ *constructive*

(2) $\Delta = \lambda/5,\quad \delta = 2\pi/5,\quad I/I_{max} = \cos^2(\pi/5) = 0.65$

(3) $\Delta = \lambda/2,\quad \delta = \pi,\quad I/I_{max} = \cos^2(\pi/2) = 0;$ *destructive*

(3) $\Delta = 3\lambda,\quad \delta = 6\pi,\quad I/I_{max} = \cos^2(3\pi) = 1;$ *constructive*

4. (25 pts) Consider a nonreflecting film of refractive index 1.38. Assume that its thickness is 9×10^{-6} cm. Calculate the wavelength (in the visible region) for which the film will be nonreflecting.

Answer:

The nonreflecting condition is achieved through destructive interference

$\delta = \pi$, or

$\Delta = \lambda / 2.$

Meanwhile, for a thin film, the path difference between the two reflective waves is

$\Delta = 2n_f d$, thus

$$d = \frac{\lambda}{4n_f}, \quad \text{or}$$

$$\lambda = 4n_f d$$

$$\lambda = 4\times1.38\times9\times10^{-6}\,\text{cm} = 4.97\times10^{-5}\,\text{cm} = 497\ \text{nm}$$