PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment - Solution

HW#6
Assignment Date: March. 16, 2010
Due Date: Mar. 23, 2010

1. (25pts) Standing waves are formed on a stretched string under tension of 1 N. The length of the string is 30 cm, and it vibrates in three loops. If the mass per unit length of the string is 0.01 g/cm, calculate the angular frequency of the vibration?

Note: one loop of the standing wave is half the wavelength. For the calculation, you need to convert the units of the parameters to the same system, either MKS (SI) or cgs.

Answer:

$$L = 3 \cdot \frac{\lambda}{2}$$

$$\lambda = \frac{2}{3}L = \frac{2}{3} \cdot 30 = 20\,\text{cm} = 0.2\,\text{m; the wavelength in MKS unit}$$

The velocity of the wave on a stretched string is known to be (see Section 11.6)

$$V = \sqrt{\frac{T}{\rho}} = \sqrt{\frac{1N}{10^{-3}\ \text{km/m}}} = \sqrt{10^3} = 31.6\ \text{m/s}$$

$$\rho = 0.01\,\text{g/cm} = \frac{0.01 \times 10^{-3}\ \text{kg}}{10^{-2}\ \text{m}} = 10^{-3}\ \text{km/m; in MKS unit}$$

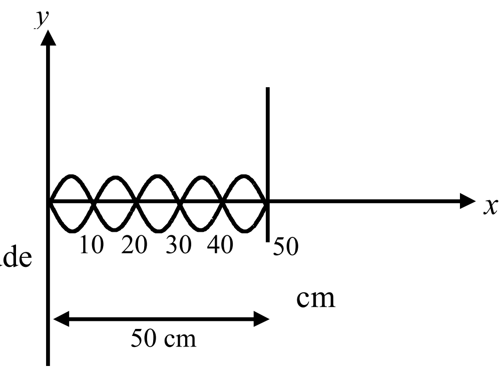
The angular frequency

$$\omega = V \cdot k = \frac{2\pi V}{\lambda} = \frac{2 \times 3.14 \times 31.6}{0.2} = 992.2\ \text{rad/s}$$

2. (25pts) Standing waves with five loops are produced on a stretched string under tension. The length of the string is 50 cm, and the frequency of vibration is 250 Hz. The amplitude of the vibration at antinodes is 2 cm.
(1) Write down the equation of the standing wave. (No derivation of the standing wave is needed, referring to the book)
(2) Derive the displacement equation of the points which are at distance of 2, 5, 10, 15, 18, 30 cm from one end of the string.

Answer:

(1) The standing wave equation on a stretched string is given as (Ch13.3, Eq. 14)



$$y(x,t) = 2a\sin\left(\frac{2\pi}{\lambda}x\right)\cos 2\pi\nu t$$

or $y(x,t) = 2a\sin(kx)\cos\omega t$

$2a$: the largest possible amplitude, or the amplitude at antinodes; 2a = 2cm

$$\mathrm{L} = 5\cdot\frac{\lambda}{2}$$

$\lambda = 20$ cm (in CGS unit)

Therefore,

$$y(x,t) = 2\sin\left(\frac{\pi}{10}x\right)\cos(500\pi t)\ \text{cm}$$

(2)

$$y(2,t) = 2\sin\left(\frac{\pi\cdot 2}{10}\right)\cos(500\pi t) = 1.18\cos(500\pi t)\ \text{cm}$$

$$y(5,t) = 2\sin\left(\frac{\pi\cdot 5}{10}\right)\cos(500\pi t) = 2.00\cos(500\pi t)\ \text{cm; anitnode point}$$

$$y(10,t) = 2\sin\left(\frac{\pi\cdot 10}{10}\right)\cos(500\pi t) = 0\ \text{cm; node point}$$

$$y(15,t) = 2\sin\left(\frac{\pi\cdot 15}{10}\right)\cos(500\pi t) = -2.00\cos(500\pi t)\ \text{cm}$$

$$y(18,t) = 2\sin\left(\frac{\pi\cdot 18}{10}\right)\cos(500\pi t) = -1.18\cos(500\pi t)\ \text{cm}$$

$$y(30,t) = 2\sin\left(\frac{\pi\cdot 18}{10}\right)\cos(500\pi t) = 0\ \text{cm; node point}$$

3. (25 pts) In Young's double-hole experiment, the distance between the two holes is 0.5 mm, $\lambda = 5 \times 10^{-5}$ cm, and D = 50 cm. What will be the fringe width?

Answer:

The interference equation is

$$\Delta = \frac{d}{D} y, \text{ or}$$

$$\mathrm{y} = \frac{D}{d} \Delta$$

For the locations of maxima, $\Delta = \mathrm{m}\lambda$, and the finger width between two consecutive orders is

$$\beta = \frac{D}{d} \lambda = \frac{50}{0.05} \times 5 \times 10^{-5} = 0.05 \text{ cm}$$

4. (25 pts) In the double-hole experiment using white light, consider two points on the projection screen, one corresponding to a path difference of 5000 Å (point A), and the other corresponding to a path difference of 40,000 Å (point B).
(1) Find all the wavelengths (in the visible region) which correspond to constructive and destructive interference at point A.
(2) Find all the wavelengths (in the visible region) which correspond to constructive and destructive interference at point B.

Answer:
The visible region corresponds to 4000 Å < λ < 7000 Å

(1) For point A, Path difference Δ = 5000 Å.
Now, for constructive interference, $\Delta = n\lambda$;
i.e., constructive interference will occur for

$$\lambda = \frac{\Delta}{n}; \quad n = 1, 2, 3, ...$$

λ = 5000 Å, 2500 Å, 1667 Å, …
Only λ = 5000 Å lies in the visible region.

Similarly, destructive interference will occur for

$$\lambda = \frac{\Delta}{n + \frac{1}{2}}; \quad n = 0, 1, 2, ...$$

λ = 10000 Å, 3333 Å, 2000 Å, …
Thus **no wavelength (in the visible region) corresponds to destructive interference**.

(2) For point B, Path difference Δ = 40000 Å.
Constructive interference will occur for

$$\lambda = \frac{\Delta}{n}; \quad n = 1, 2, ...$$

λ= 40000 Å, 20000 Å, 13333 Å, 10000 Å, 8000 Å
6667 Å, 5714 Å, 5000 Å, 4444 Å, 4000 Å, 3636 Å, …

Thus λ = 6667 Å, 5714 Å, 5000 Å, 4444 Å and 4000 Å (which lie in the visible region) would have constructive interference.

Destructive interference will occur for

$$\lambda = \frac{\Delta}{n + \frac{1}{2}}; \quad n = 0, 1, 2, ...$$

λ = 6154 Å, 5333 Å, 4706 Å, 4211 Å, …
for n = 6, 7, 8 and 9 respectively.

Therefore, the color of fringe at point A is yellow. At point B, the color would be white. The fringe pattern almost disappears because of the mix of multiple colors as constructive, and at the same location the mix of multiple colors as destructive.