PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Assignment

HW#4
Assignment Date: Feb. 16, 2010
Due Date: Feb. 23, 2010

1. (20pts) The displacement associated with a wave is given by

$$y(x,t) = 3.0\cos(0.5x - 2t)$$

where x and y are measured in meters and t in seconds.
(1) what is the spatial frequency? and what is the angular frequency?
(2) calculate the wavelength, period, (linear) frequency and wave velocity

Answer:
(1). Spatial frequency: k = 0.5 m^{-1}
Angular frequency: ω = 2 s^{-1}
(2)

$$\text{From } k = \frac{2\pi}{\lambda},\ \text{wavelength } \lambda = \frac{2\pi}{k} = 12.6\ \text{m}$$

$$\text{From } \omega = \frac{2\pi}{T},\ \text{period } T = \frac{2\pi}{\omega} = 3.1\ \text{s}$$

$$\text{Linear frequency } \upsilon = \frac{\omega}{2\pi} = \frac{1}{T} = 0.32\ s^{-1}\text{, or } 0.32\ \text{Hz}$$

$$\text{Wave velocity v} = \frac{\omega}{\text{k}} = \frac{\lambda}{T} = 4.0\ \text{m s}^{-1}$$

The wave moves along + x direction, since as time t goes,
x needs to increase to keep the same phase of the wave

2. (20 pts) The displacement associated with a wave is given by

$$y(x,t) = 0.5\cos(6x+3t)$$

where x and y are measured in meters and t in seconds.

(1) what is the spatial frequency? and what is the angular frequency?

(2) calculate the wavelength, period, (linear) frequency and wave velocity

Answer:

(1). Spatial frequency: $k = 6\ m^{-1}$

Angular frequency: $\omega = 3\ s^{-1}$

(2)

$$\text{From } k = \frac{2\pi}{\lambda},\ \text{wavelength } \lambda = \frac{2\pi}{k} = 1.05\ \text{m}$$

$$\text{From } \omega = \frac{2\pi}{T},\ \text{period } T = \frac{2\pi}{\omega} = 2.09\ \text{s}$$

$$\text{Linear frequency } \upsilon = \frac{\omega}{2\pi} = \frac{1}{T} = 0.48\ s^{-1},\ \text{or } 0.48\ \text{Hz}$$

$$\text{Wave velocity v} = \frac{\omega}{\text{k}} = \frac{\lambda}{T} = 0.5\ \text{m s}^{-1}$$

The wave moves along – x direction, since as time t goes, x needs to decrease in order to keep the same phase of the wave

3. (20 pts) A Gaussian pulse is propagating in the -x (minus x) direction, and at $t=t_0$ the displacement is given by

$$y(x, t=t_0) = a\exp[-\frac{(x-b)^2}{\sigma^2}]$$

Find y(x,t)

Answer

:

First, move along - x, x shall be replaced by x + vt

Second, t is replaced by t - t_0, in order to recover the original wave form \

$$y(x,t) = a\exp[-\frac{[x+v(t-t_0)]-b)^2}{\sigma^2}]$$

4. (40 pts) Consider a wave propagating in the +x direction whose (linear) frequency is 10 s^{-1}. The medium of the wave has a mass density of 0.1 g cm^{-3}. At t =5 s the displacement associated with the wave is given by

$$y(x,t=5) = 0.5\cos(0.2x)$$

where x and y are measured in centimeters and t in seconds

(1) Find y(x,t)

(2) Obtain the displacement (as a function of x) at t=10 s

(3) What are the wavelength and velocity associated with the wave?

(4) What is the wave energy density?

(5) What is the wave intensity?

Answer: First note that the unit used here is CGS, not SI (or MKS)

(1)

$$\omega = 2\pi \cdot \nu = 62.8\,\mathrm{s}^{-1}$$

$$y(x,t) = 0.5\cos[0.2x - 62.8(t-5)]$$

(2)

$$y(x,t=10) = 0.5\cos[0.2x - 62.8(10-5)]$$

$$y(x,t=10) = 0.5\cos[0.2x - 314.0]$$

(3)

$$\lambda = 2\pi / k = 31.4\,\mathrm{cm}$$

$$\mathrm{v} = \frac{\omega}{k} = \frac{62.8}{0.2} = 314.0\,\mathrm{cm/s}$$

(4)

Wave energy density is given by

$$\varepsilon = \frac{1}{2}\rho a^2 \omega^2 = 49.3\,\mathrm{erg/cm}^3$$

(4)

Wave energy Intensity is given by

$$I = \varepsilon \cdot V = 1.55 \times 10^4\ \mathrm{erg\ cm^{-2}\ s^{-1}}$$