PHYS 306 Spring 2010
Wave Motion and Electromagnetic Radiation

Homework Solution

HW#1
Assignment Date: Jan. 26, 2010
Due Date: Feb. 02, 2010

1. (Problem 7.3 in Ghatak book). A tunnel is dug through the earth as shown in Fig. 7.15. A mass is dropped at the point A along the tunnel. Show that it will execute simple harmonic motion. What will the time period be?

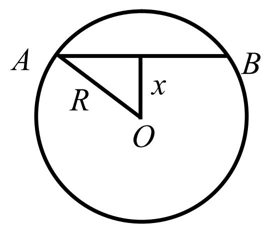
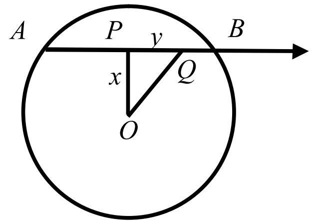


Fig. 7.15 For Problem 7.3

Answer (30 pts):



As shown in the Figure above, the particle moves along the "y" direction (AB, positive toward B). The only force along the "y" is the centripetal gravitational force projected on "y". This force is expected to cause the harmonic oscillation of the particle. Considering the point Q, which is at y, and distance r from the center

$$F_y = -mg_r \frac{y}{r}$$

$$g_r = \frac{GM_r}{r^2} = \frac{G\frac{4}{3}\pi\rho r^3}{r^2}$$ in which constant density with r is assumed

$$g_r = \frac{4G}{3}\pi\rho r$$

In terms of the surface gravity g at the surface R

$$\frac{g_r}{g} = \frac{r}{R}$$

$$F_y = -mg\frac{r}{R}\frac{y}{r} = -mg\frac{y}{R}$$

Therefore, the EOM is

$$\mathrm{m}\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2} = F_y = -mg\frac{y}{R}$$

$$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2} + \frac{g}{R}y = 0$$

Compared with the stardard EOM of the simple harmonic motion, we know that general solution is $\mathrm{y} = \mathrm{Acos}(\omega \mathrm{t} + \phi)$, with

$$\omega = \sqrt{\frac{g}{R}}$$

$$\mathrm{T} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$$

2. (Problem 7.4 in Ghatak book). A 1 g mass is suspended from a vertical spring. It executes simple harmonic motion with period 0.1 sec. By how much distance had the spring stretched when the mass was attached?

Answer (30 pts):

Use the period of harmonic motion to determine the force constant of the spring, then use the Hooke's law of elasticity to determine the displacement.

The EOM is

$$m\frac{d^2x}{dt^2} = -kx,$$

where x is the displacement from the equibrium position when the mass is attached, and k is the force constant

$$\Rightarrow \frac{d^2x}{dt^2} + \omega_0^2\, x(t) = 0; \quad \omega_0 = \sqrt{\frac{k}{m}}$$

Thus $T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}}$. If $T = 0.1$ s, $m = 1$ g, $k = 400\pi^2 \approx 3.95\times10^3$ dynes/cm

Using Hooke's law

$$k\Delta x = mg,$$

where Δx is the displacement when the mass is attached from that the mass is not attached

$$\Rightarrow \Delta x = \frac{mg}{k} = \frac{980 \text{ g.cm/s}^2}{3.95\times10^3 \text{ dynes/cm}} \approx 0.25 \text{ cm}$$

The CGS (cm - gram - sec) unit is used in the calculation above.

Similary, one could use SI or MKS (meter - kilogram - sec) unit.

3. In the simplest model of the atom, the electrons are assumed to be bound elastically to their rest position. The equation of motion for the electron, in the presence of an external electric field E and assuming zero damping, would be

$$m\frac{d^2\vec{x}}{dt^2} + k_0\vec{x} = -q\vec{E}$$

(1) Show that the refractive index n of a dielectric medium is

$$n^2 = 1 + \frac{Nq^2}{m\varepsilon_0\omega_0^2}(1 - \frac{\omega^2}{\omega_0^2})^{-1}$$

(2) Derive the Cauchy relation

$$n^2 = A + \frac{B}{\lambda_0^2}$$

Note: refer to CH7.5 of Ghatak book. Adopt the convention of symbols, and also use the assumptions provided in the book.

Answer (40 pts): Refer to the Gatak book P. 103 - 104 for the detailed answer.