Nov. 18, 2010

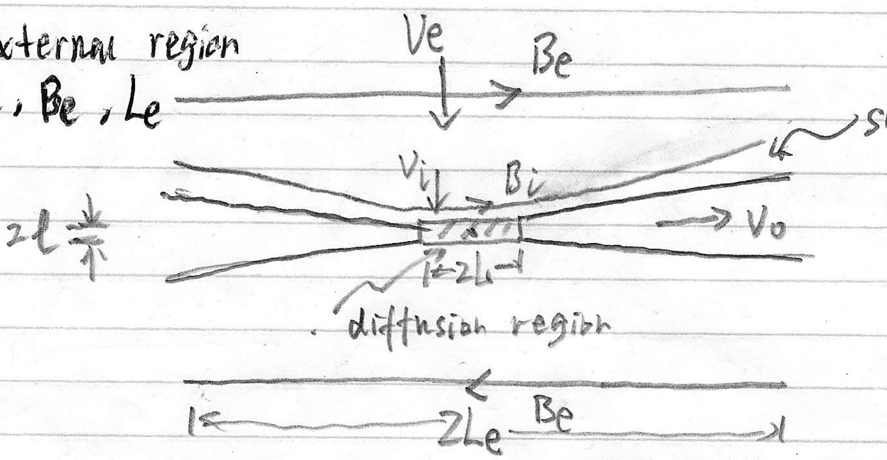
Petschek's Mechanism of Fast Reconnection (CH4.3)

- From great physical Insight
- Not from mathematical rigor

In addition to magnetic dissipation in current sheet,
magnetic dissipation occurs in slow-mode MHD shocks.

MHD shock waves — CH 1.5. ~~(explained later)~~.

① * External region
V_e, B_e, L_e



Ve = external speed
Vi : inflow speed
Be : enternal $\vec{B}$
Bi : inflow $\vec{B}$
Le: global size

Petschek's model (4.4), (4.6)

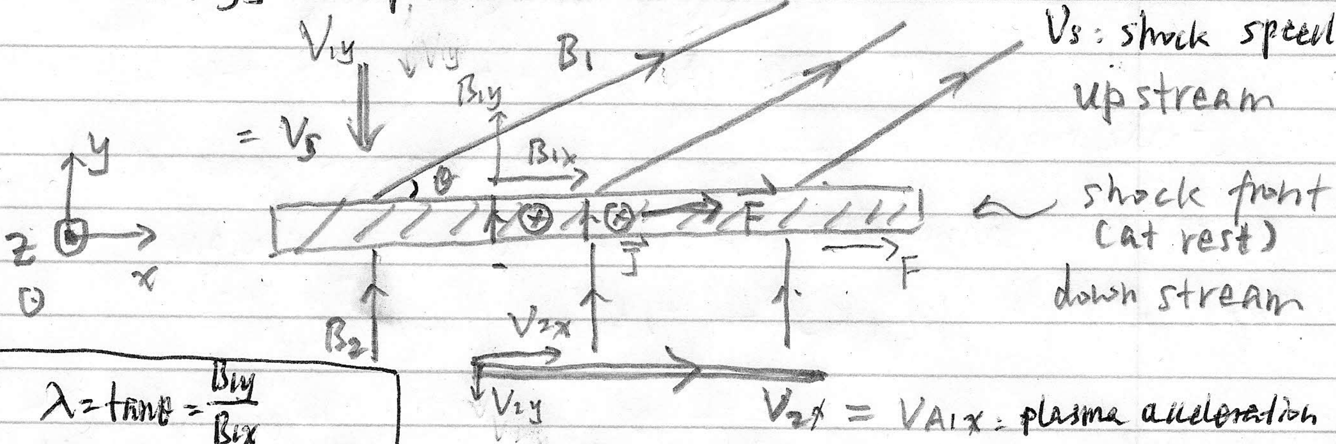
② * Diffusion region is shortened L, instead of L_e. (l, L)
$L << L_e$.
L_e: global length scale
l is also smaller than in Sweet-Parker
but l/L is the same as in Sweet-Parker

③ * Inner ~~Inflow~~ region: V_i, B_i

* Four slow-mode MHD shock form.
  extending out from the four corners of diffusion region
  surrounding the outflow region
  dissipating the inflow magnetic energy
* $V_i > V_e$, $B_i < B_e$.
  Field is curved

(Continued)

\# energy dissipation in slow-mode MHD shock



$$\boxed{\begin{aligned}\lambda &= \tan\theta = \frac{B_{1y}}{B_{1x}} \\ V_s = V_{1y} &= \frac{B_{1y}}{(\mu\rho)^{\frac{1}{2}}} = \frac{\lambda V_{A1}}{\sqrt{1+\lambda^2}}\end{aligned}}$$

Fig. 4-5 (b) (Note: Coordinate x–Y is different from CH 1-5)

standing shock front acts as an obstacle in the flow

① deflect the magnetic field, become normal

$$\nabla \cdot \vec{B} = 0 \qquad \oint_S \vec{B} \cdot d\vec{S} = 0$$

$$\Rightarrow \quad B_{1n} = B_{2n}, \text{ or } \boxed{B_{1y} = B_{2y}}$$

The $\vec{B}$ normal component is continuous

② Tangential component of $\vec{B}$ dissipated

$$B_{1x} = \cancel{B_1 \sin\theta}\ B_1 \cos\theta$$

$\boxed{B_{2x} = 0}$; $\vec{B}_2$ has only normal component $\Rightarrow$ $\boxed{\text{switch-off shock}}$

③ plasma is accelerated along X-direction

$$V_{2x} = V_{A1x} = \frac{V_{A1}}{\sqrt{1+\lambda^2}}$$

$\vec{J} = \nabla \times B$ $\quad$ $\vec{J}$: $-\hat{z}$ direction

$\vec{F} = \vec{J} \times \vec{B}$. $\quad$ $\vec{F}$: $+\hat{x}$ direction

$\Rightarrow$ Magnetic energy is converted to plasma kinetic energy $(\frac{3}{5})$ and thermal energy $(\frac{2}{5})$

$\gamma = \frac{5}{3}$

(3)

(Continued)

* Derive M_e in Petschek's model

M_e: dimensionless reconnection rate

* First, relation the three regions of L_e, L, ℓ

External Region Size : L_e

Diffusion-Region Length : L, size of inner region

Diffusion-Region width : ℓ.

$$\frac{\ell}{L} = \frac{1}{\sqrt{M_i}}$$: the same as in Sweet-parker mechanism

How about L and L_e?

$$R_{Me} = \frac{L_e V_{Ae}}{\eta} \Rightarrow L_e = \frac{\eta R_{Me}}{V_{Ae}}$$

$$R_{Mi} = \frac{L V_{Ai}}{\eta} \Rightarrow L = \frac{\eta R_{Mi}}{V_{Ai}}$$

$$\frac{L}{L_e} = \frac{R_{Mi}}{R_{Me}} \cdot \frac{V_{Ae}}{V_{Ai}}$$: To Find dimensionless expression; --- (A)

* relate V_i and V_e

using $\vec{E}$ = constant in 2-D steady-state flow

$$V_e B_e = V_i B_i$$

$$V_{Ae} = \frac{B_e}{\sqrt{\mu\rho}}$$

$$V_{Ai} = \frac{B_i}{\sqrt{\mu\rho}}$$

$$\frac{M_i}{M_e} = \frac{\frac{V_i}{V_{Ai}}}{\frac{V_e}{V_{Ae}}} = \frac{V_i}{V_e} \cdot \frac{V_{Ae}}{V_{Ai}} = \frac{B_e}{B_i}\frac{B_e}{B_i}$$

$$\Rightarrow \boxed{\frac{M_i}{M_e} = \frac{B_e^2}{B_i^2}} \quad \text{---- (4.36)}$$

We know $\boxed{M_i = \frac{1}{\sqrt{R_{Mi}}}}$ - From Sweet-Parker.

$$R_{Mi} = M_i^{-2}, \quad \frac{V_{Ae}}{V_{Ai}} = \frac{B_e}{B_i} = \frac{M_i^{\frac{1}{2}}}{M_e^{\frac{1}{2}}}$$ plug-in to (A)

$$\frac{L}{L_e} = \frac{M_i^{-2}}{R_{me}} \cdot \frac{M_i^{\frac{1}{2}}}{M_e^{\frac{1}{2}}}$$

$$\Rightarrow \boxed{\frac{L}{L_e} = \frac{1}{R_{me}} \, \frac{1}{M_e^{\frac{1}{2}}} \, \frac{1}{M_i^{\frac{3}{2}}}} \quad \cdots \quad (4.37)$$

To solve the system and define V_e (or M_e)

L_e, B_e is known

[The key is to find B_i]

Once B_i is known, $\frac{M_i}{M_e}$ is known

L is known in terms of M_e

or M_e is known in terms of L.

$M_e \uparrow$; $L \uparrow$

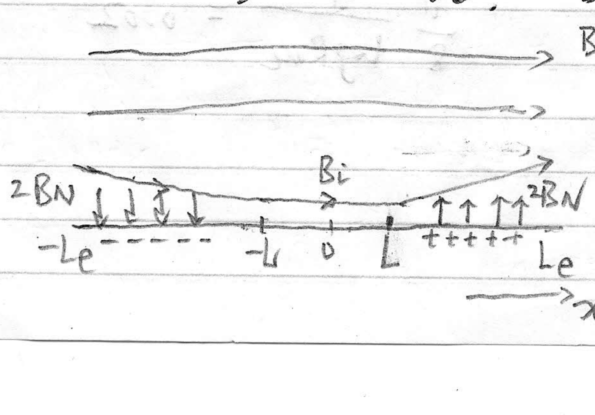
Fast inflow requires smaller L.

[smaller diffussion region $\rightarrow$ fasterreconnection rate]

* Second step is to find B_i

The effect of shock is to provide a normal field component B_N which is associated with the distortion in the inflow field from uniform value B_e

$B_i < B_e$. $V_i > V_e$. $B_N > 0$ due to distortion



B_N can be regarded as produced by "line monopole" "m" between L and L_e

"m": line density of the monopole

Far field $B_r = \frac{m(L_e-L)}{r}$

Total far field flux $Flux = \int_0^{\pi} \frac{m(L_e-L)}{r}\cdot r\,d\theta = \pi m(L_e-L)$

Near field flux caused by $2B_N$. $Flux = 2B_N(L_e-L)$

$$\Rightarrow m = \frac{2B_N}{\pi}$$

This monopole reduce $\vec{B}$ in the diffusion region causing $B_i < B_e$

$$\Delta B = B_e - B_i = 2\cdot\int_L^{L_e} \frac{m\,dx}{x} = \frac{4B_N}{\pi}\int_L^{L_e}\frac{1}{x}\,dx$$

$$\boxed{B_i = B_e - \frac{4B_N}{\pi}\log\frac{L_e}{L}} \quad \text{---- } (4.40)$$

$$B_i = B_e\left(1-\frac{4}{\pi}\frac{B_N}{B_e}\log\frac{L_e}{L}\right)$$

We know, for standing slow-mode shock

$V_s = V_e$. shock speed = inflow speed

$$V_s = \frac{B_N}{\sqrt{\mu\rho}}$$

$$\Rightarrow \frac{B_N}{B_e} = \frac{V_s}{V_{Ae}} = \frac{V_e}{V_{Ae}} = M_e$$

$$\Rightarrow B_i = B_e\left(1-\frac{4M_e}{\pi}\log\frac{L_e}{L}\right) \quad \text{-- } (4.41)$$

Petschek estimate the maximum reconnection M_e^* occurs at $B_i/B_e = \frac{1}{2}$; Too small will stop the process

$$M_e \approx \frac{\pi}{8}\cancel{\log L_e}\frac{1}{\log(\frac{L_e}{L})} = \frac{\pi}{8}\frac{1}{\log R_{me} + \underbrace{\log(M_e^{\frac{1}{2}} M_e^{\frac{3}{2}})}_{\text{extremely small}}}$$

$$\Rightarrow \boxed{M_e^* \approx \frac{\pi}{8\log R_{me}}} \approx 0.1 \sim 0.01 \quad (4.43)$$

New Regimes of fast reconnection — CH 5.1

A robust ~~generalition~~ generalization of 2-D ~~solution~~ reconnection through analytic solution

$$\vec{B} = \vec{B_e} + M_e \vec{B_1}$$; similar to Taylor expansion,

$$\vec{V} = M_e \vec{V_1}$$ Keep the first order

Solving MHD equations, one can find

$$\left(\frac{M_e}{M_i}\right)^2 \approx \frac{4M_e(1-b)}{\pi}\left[0.834 - \log\tan\left(\frac{4R_{me}M_e^{\frac{1}{2}}M_i^{\frac{1}{2}}-1}{\pi}\right)\right]$$

The reconnection rate is sensitive to "b" number.

Magnetic field line and flow line also sensitive to "b"

---- : $\vec{V}$ stream line

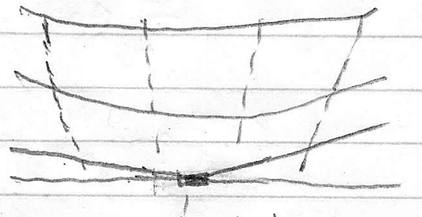
—— : $\vec{B}$ line

b=0. Petschek regime

converging motion of $\vec{V}$

$V\uparrow$, $P\downarrow$

$B\downarrow$



short L

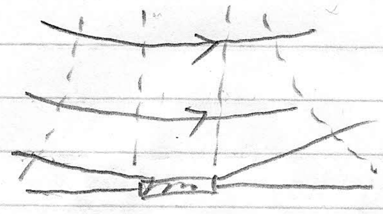
P and B in phase. called fast mode expansion

b=1. Sonnerup regime

diverging motion of $\vec{V}$

$\vec{V}\uparrow$, $P\downarrow$

$\vec{B}$ almost constant



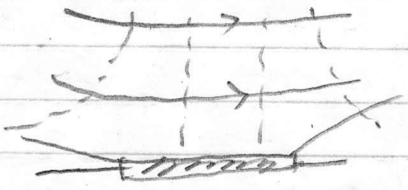
Long L

Refer to Fig 5.4

b>1 flux pile-up

diverging motion of $\vec{V}$

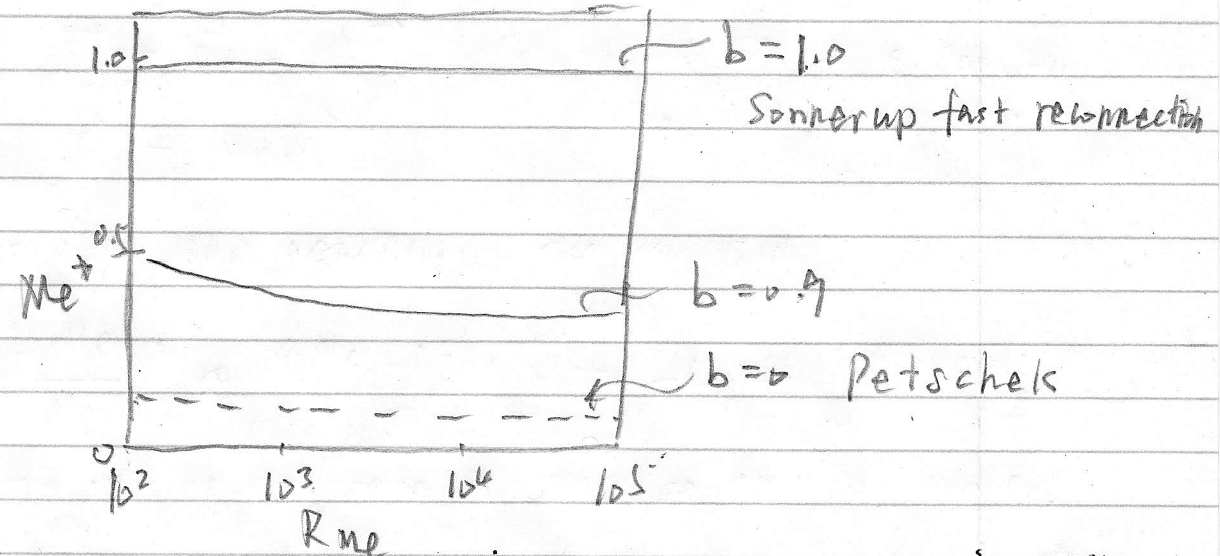
$\vec{V}\uparrow$. $P\downarrow$



very long L

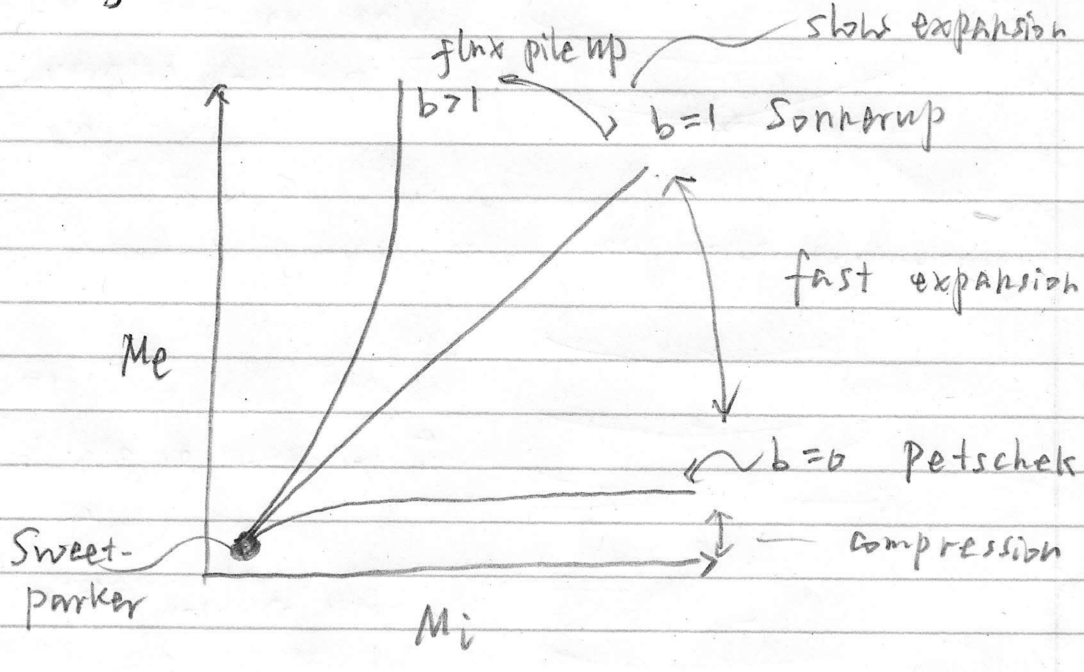
$\vec{B}\uparrow$, pile up ; P, $\vec{B}$ not in phase, slow-mode expansion

Fig 5.2. Reconnection rate (Me) as a function of Rme for different b value



b increases ⇒ reconnection increases
Me ~ 1

Fig 5.3. Me — M_i plot



M_i: inflow speed in internal region
Me: external speed in external region

Overview of chap 6, chap.7, chap 8.

chap. 6 : Unsteady reconnection : tearing mode.

Instability of current sheet

→ formation of magnetic island.

and then magnetic energy dissipates faster

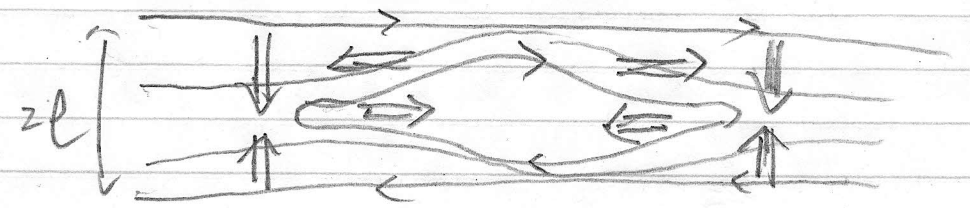


Fig 6-3 : flow field ~~(dotted)~~ double arrow and $\vec{B}$ field

Chap. 7. Unsteady Reconnection : X-Type collapse

How to describe or mode the collapse process?

☆ Chap. 8. Reconnection in Three Dimension

* Three Dimension null point
* Spine and fan
* magnetic flipping

Recommend to read and study chap. 8 on your own

chap. 9 — 12 : applications

☆ chap. 13. particle acceleration