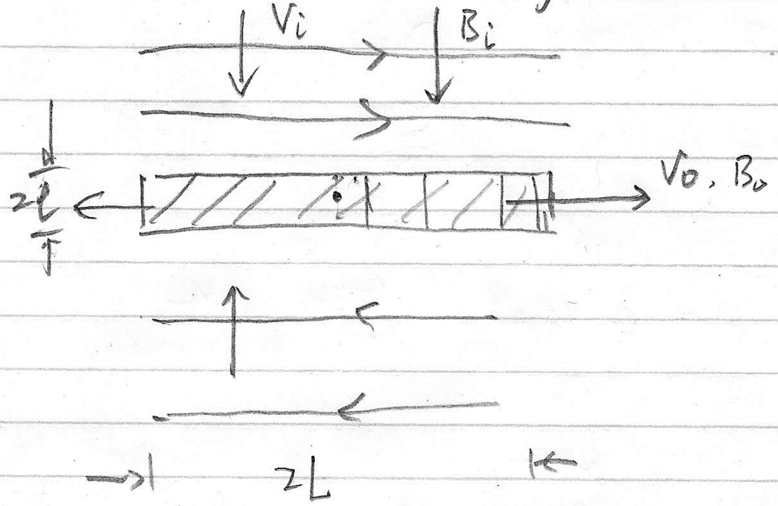
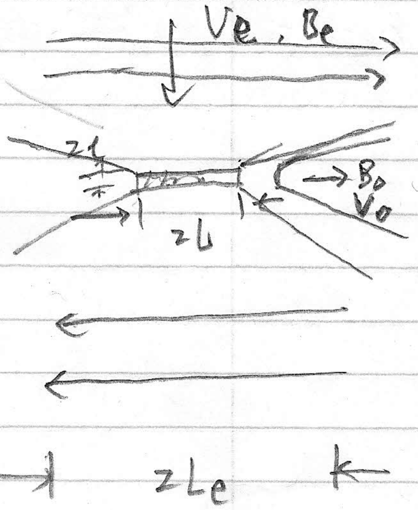
Oct 28, 2010

Sweet-Parker Mechanism (CH 4.2) (1957)

Consider the diffusion region



Global view



Find the steady state solution : order of magnitude analysis

find V_i, V_o, l for given L

Ohm's Law: $\vec{E} + \vec{v} \times \vec{B} = \frac{\vec{j}}{\sigma}$

$\vec{E}$: constant

Outside diffusion region $\vec{j} \to 0$

$\vec{E} = -\vec{v} \times \vec{B}$

$E = V_i B_i$, $E = V_o B_o$

V_i: inflow speed

V_o: outflow speed

Inside the diffusion region $\vec{B} \to 0$

$E = \frac{j_D}{\sigma}$ j_D, average current in diffusion region

$j_D = \frac{B_i}{\mu l}$ ——— (4.6)

$\Rightarrow V_i B_i = \frac{B_i}{\sigma \mu l} \Rightarrow \boxed{V_i = \frac{\eta}{l}}$ ——— (4.9)

$V_i = \frac{\eta}{l}$ also called $\boxed{\text{reconnection rate}}$, that is how fast magnetic field diffuses across the plasma.

(continued)

Conservation of mass

$$\underbrace{L V_i}_{\text{mass in}} = \underbrace{l V_o}_{\text{mass out}}$$

To eliminate l

$$V_i = \frac{\eta}{l} = \frac{\eta}{\frac{V_i}{V_o} L} \Rightarrow \boxed{V_i^2 = \frac{\eta V_o}{L}} \quad \text{--- (4.12)}$$

V_o ?

☆ Prove $V_o = V_{Ai} \equiv \frac{B_i}{\sqrt{\mu\rho}}$ = inflow ~~plasma~~ Alfven speed

the outward velocity V_o is accelerated by the Lorentz force along the current sheet

$$j = \frac{B_i}{\mu l} \quad : \text{current inside}$$

$$\text{Lorentz force } (\vec{j} \times \vec{B})_x = j B_o = \frac{B_i B_o}{\mu l}$$

From momentum equation

$$\rho (\vec{V} \cdot \nabla) \vec{V} = -\underbrace{\nabla p}_{=0} + \vec{j} \times \vec{B}$$

$$\Rightarrow \rho \frac{V_o^2}{L} = \frac{B_i B_o}{\mu l} \quad \text{--- (4.18)}$$

Since mass conservation $L V_i = l V_o$

E : constant $\quad V_i B_i = V_o B_o$ — flux-conservation

$$\Rightarrow \frac{B_o}{l} = \frac{B_i}{L} \qquad B_o \ll B_i \quad \text{--- (4.19)}$$

(4.18) + (4.19) $\Rightarrow V_o^2 = \frac{B_i^2}{\mu\rho} \Rightarrow \boxed{V_o = \sqrt{\frac{B_i^2}{\mu\rho}} = V_{Ai}}$ --- (4.20)

(4-20) → (4.12)

$$V_i^2 = \frac{\eta V_{Ai}}{L}$$

(continued)

Use Reynold number notation to replace L

$$R_{mi} = \frac{L V_{Ai}}{\eta} \quad \text{------} \quad (4.15)$$

Repeat

$$V_i^2 = \frac{\eta \cdot V_{Ai}}{R_{mi} \eta \frac{1}{V_{Ai}}} = \frac{V_{Ai}^2}{R_{mi}}$$

$$\boxed{V_i = \frac{V_{Ai}}{\sqrt{R_{mi}}}} \quad \text{------} \quad (4.21)$$

$$l = L \frac{V_i}{V_0} = L \frac{V_i}{V_{Ai}} \Rightarrow \boxed{l = \frac{L}{\sqrt{R_{mi}}}} \text{ --}$$

$$\boxed{B_0 = \frac{B_i}{\sqrt{R_{mi}}}}$$

$$\boxed{V_0 = V_{Ai}}; \quad M_i = \frac{V_i}{V_{Ai}} = \frac{1}{\sqrt{R_{mi}}}$$

In sweet-parker mechanism.

L is assumed to be global length L_e

e : external

$L = L_e$.

$V_{Ai} = V_{Ae}$.

$$R_{mi} = R_{me} = \frac{L_e V_{Ae}}{\eta}$$

In corona.

$$M_e = M_i = \frac{1}{\sqrt{R_{me}}} \quad : \text{dimensionless reconnection rate}$$

If $R_{me} = 10^8$, $M_e = 10^{-4}$.

too slow for solar flare.

need $M_e \sim 10^{-1}$.

Reconnection rate: (CH 1.6)

how fast magnetic field is reconnected at null point.

which is how fast magnetic stream line changes?

In 2-D field; 2-D flow (V_x, V_y), 2-D $\vec{B}$ (U_x, U_y)

$$\vec{E} = E_{(x,y)}\hat{z}, \quad E_{(x,y)} = \text{const.}$$

Further $\vec{B} = \nabla \times \vec{A}$, $\vec{A}$: vector potential

$$\vec{A} = A_{(x,y)}\hat{z}.$$

$A(x,y)$ is the stream line function

From Faraday's Law: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial(\nabla \times A)}{\partial t}$$

$$\Rightarrow \frac{\partial \vec{A}}{\partial t} = -\vec{E} + \nabla\phi$$

Therefore, $\boxed{\text{reconnection rate } \left(\frac{\partial \vec{A}}{\partial t}\right)}$ is $\boxed{\text{determined by } \vec{E}}$

For a 2-D flow, Generalized Ohm's Law

$$\vec{E} = \frac{\vec{J}}{\sigma} - \vec{v} \times \vec{B}$$

At the boundary of the flow system: V_e, B_e. e: external

$$\vec{j} = 0, \quad E = V_e B_e$$

Therefore, reconnection rate is determined $\boxed{\text{by } V_e}$ for given B_e

Further, introduce $\boxed{\text{dimensionless reconnection rate}}$

$$M_e = \frac{V_e}{V_{Ae}}$$

M_e: Alfven Mach number

V_{Ae}: Alfven speed $\quad V_{Ae} = \sqrt{\frac{B^2/\mu_0}{\rho}} = \sqrt{\frac{\text{Magnetic Pressure}}{\text{density}}} = \frac{B_e}{\sqrt{\mu_0 \rho_e}}$

similarly, sound speed $V_s = \sqrt{\frac{\gamma P}{\rho}}$, P: thermal pressure

\# (continued)

* 1-D stagnation-flow model :

$$M_e \sim \frac{1}{R_{me}} \Rightarrow \text{extremely slow magnetic reconnection}$$

Both M_e, R_{me} are dimensionless

In corona, $R_{me} \sim 10^8$

$M_e \sim 10^{-8}$

* Sweet-parker mechanism

$$M_e \sim \frac{1}{\sqrt{R_{me}}}$$

Faster than 1-D stagnation model

$M_e \sim 10^{-4}$, still slow

$\Rightarrow$ slow magnetic reconnection

* Fast reconnection

To explain solar flares, need $M_e \sim 10^{-1}$

$\Rightarrow$ fast reconnection

e.g. $M_e \sim \dfrac{1}{\log R_{me}}$

Petschek reconnection model

\# Energy Consideration in S-P model (CH 4.2.3)

Inflow EM energy: $\underbrace{\frac{\vec{E}\times\vec{B}}{\mu_0}}_{\text{Poynting flux}} \cdot V_i L = \frac{E B_i}{\mu_0} V_i L$

$= V_i \underbrace{\frac{B_i^2}{\mu_0}}_{\text{magnetic energy density}} L$ since $E = V_i B_i$

Inflow KE: $\frac{1}{2}\rho V_i^2 \cdot V_i L$

$$\frac{\text{Inflow KE}}{\text{Inflow EM}} = \frac{\frac{1}{2}\rho V_i^2}{B_i^2/\mu_0} = \frac{V_i^2}{2V_{Ai}^2} = \frac{1}{2R_{mi}} << 1$$

Thus, most inflowing energy is magnetic.

$$\frac{\text{Outflow EM}}{\text{Inflow EM}} = \frac{\cancel{V_o l}\cdot\frac{B_o^2}{\mu_0}}{\cancel{V_i L}\cdot\frac{B_i^2}{\mu_0}} = \frac{B_o^2}{B_i^2} = \frac{1}{R_{mi}} << 1$$

Outflow EM energy is negligible ~ 0

* Question: Where does the inflow magnetic energy go?

Answer: half kinetic energy of outflow jet plasma
half of Joule heating of plasma

$$\frac{\text{outflow KE}}{\text{Inflow EM}} = \frac{\frac{1}{2}\rho V_o^2 (V_o l)}{\frac{B_i^2}{\mu_0}\cancel{V_i L}} = \frac{1}{2}\frac{\cancel{V_{Ai}^2}}{\underbrace{\frac{B_i^2}{\rho\mu_0}}\cancel{V_{Ai}^2}} = \frac{1}{2}$$