Oct. 21, 2010 (1)

Advection : (CH 3. 1-2).

magnetic accumulation caused by motion

Assume $R_m \gg 1$, no diffusion, and frozen-in

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B})$$

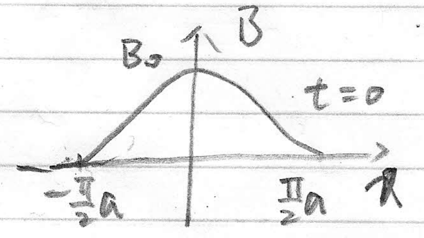
Again, 1-D magnetic field

but $\vec{V}$: 2D flow

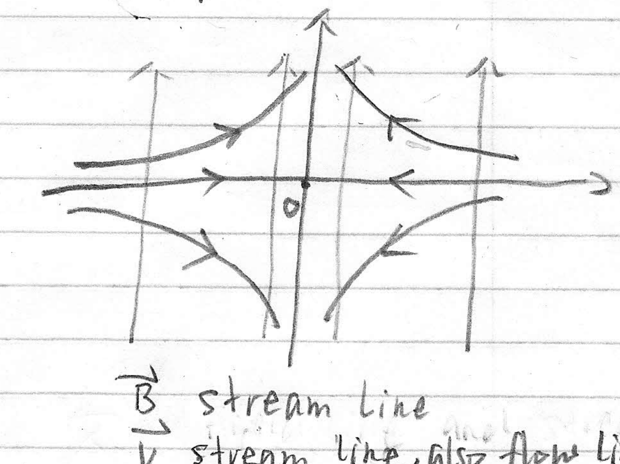
Assume
$$\begin{cases} V_x = -\dfrac{V_0}{a} x \\ V_y = \dfrac{V_0}{a} y \end{cases}$$

$$B(x, t=0) = B_0 \cos\left(\frac{x}{a}\right) \hat{y} \quad \text{at } t=0$$

a : the scale-size of the $\vec{B}$



$\vec{B}$ plot



$\vec{B}$ stream line

$\vec{V}$ stream line, also flow line

At 0 : stagnation point

because $V_x = 0$, $V_y = 0$ (versus null point $\vec{B} = 0$)

Field line : always straight, since V_x = const for all y

stream line : $\dfrac{V_x}{dx} = \dfrac{V_y}{dy} \Rightarrow \dfrac{-x}{dx} = \dfrac{y}{dy}$

$-\ln x = \ln y + c \Rightarrow xy = \text{const}$

hyperbola

$\Rightarrow$ looking for $B(x,t)$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \quad , \quad \vec{B} \text{ only } B(x)\,\hat{y}$$

$$\Rightarrow \frac{\partial B}{\partial t} = -\frac{\partial}{\partial x}(V_x B)$$

$$\frac{\partial B}{\partial t} - \frac{V_0 x}{a}\frac{\partial B}{\partial x} = \frac{V_0}{a} B$$

* consider the time variation only

$$\frac{dB}{dt} = \frac{V_0}{a} B$$

$$\frac{dB}{B} = \frac{V_0}{a} dt \Rightarrow \ln B = \frac{V_0 t}{a} + c$$

$$\Rightarrow B = B_0 e^{\frac{V_0 t}{a}}$$: exponentially increase with time

* Consider the position change only.

Because of frozen-in effect, $\vec{B}$ follows $\vec{v}$

$$v_x = -\frac{V_0 x}{a}$$

$$\Rightarrow \frac{dx}{dt} = -\frac{V_0 x}{a} \Rightarrow x = x^* e^{-\frac{V_0 t}{a}} \; ; \; x^* \text{: initial position}$$

called Characteristic curve

$$\Rightarrow B_{(x,t)} = B_0 \cos\left(\frac{x^*}{a}\right) e^{\frac{V_0 t}{a}}$$

$$B_{(x,t)} = B_0 \cos\Big(\underbrace{\frac{x}{a} e^{\frac{V_0 t}{a}}}_{\text{width}}\Big) \underbrace{e^{\frac{V_0 t}{a}}}_{\text{amplitude}}$$

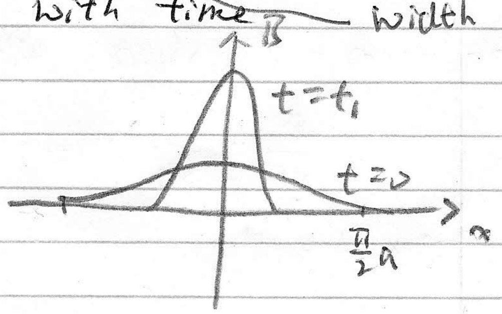
$\vec{B}$: exponential increase with time

$t\uparrow$ $B\uparrow$

accumulation

Total flux is conserved

$$\Phi = \int_{-\infty}^{\infty} B(x,t)\,dx = B_0 a$$



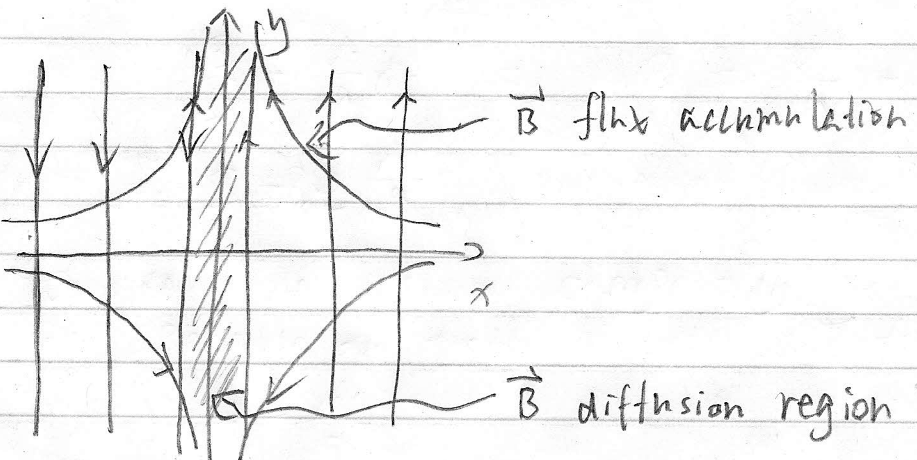
\# Stagnation - Point Flow Model (CH 3-2)

1. consider flux accumulation due to advection
2. consider flux dissipation due to resistivity and large current

$\Rightarrow$ Reach a steady state of $\vec{B}$, $\vec{J}_z$, $\frac{\partial \vec{B}}{\partial t} = 0$.

Flow field $\begin{cases} V_x = -\frac{V_0 x}{a} \\ V_y = \frac{V_0 y}{a} \end{cases}$

~~Init~~ $\vec{B}$: symmetric, but opposite direction in $x>0$ and $x<0$



$\vec{B}$ stream line

$\vec{V}$ stream line, flow line

Solve the MHD equation : rare case an analytic solution exists

$$\vec{E} + \vec{V} \times \vec{B} = \eta \nabla \times \vec{B} \quad \text{------} \quad (3.11)$$

Is Ohm's Law : $\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B})$

Is Integration of magnetic induction equation

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) + \eta \nabla B^2$$

$\vec{V}$, $\vec{B}$ only have $\hat{x}$, $\hat{y}$ component, $\vec{V} \times \vec{B}$: in $\hat{z}$ direction

$\nabla \times (B_{(x)} \hat{y}) = (\underbrace{-\frac{\partial B_y}{\partial y}}_{=0}) \hat{x} + (\frac{\partial B_y}{\partial x}) \hat{z}$, also only $\hat{z}$ component

(Continued)

In 2-D flow and magnetic field, $\vec{E}$ must be in $\hat{z}$ direction

Since $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$ [Faraday's Law, steady state]

$$\Rightarrow \frac{\partial E_z}{\partial x} = 0 \quad \text{and} \quad \frac{\partial E_z}{\partial y} = 0$$

$\Rightarrow E_z$ is constant everywhere in x-y plane, along $\hat{z}$ direction

The generalized Ohm's Law reduce to

$$E - \frac{V_0 x}{a} B = \eta \frac{dB}{dx} \quad \text{--- (3.15)}$$

General solution exists

$$B = \frac{2Ea}{V_0 l} \operatorname{daw}\left(\frac{x}{l}\right) ; \quad l = \sqrt{\frac{2\eta a}{V_0}}$$

Dawson integral function $\operatorname{daw}(X) = \exp(-X^2)\int_0^X \exp(t^2)\,dt$

$$\operatorname{daw}(X) \simeq \begin{cases} \frac{1}{2X} & X \gg 1 \quad \text{far field} \\ X & X \ll 1 \quad \text{near field} \end{cases}$$

The approximation can be found from (3.15)

If x is small, (3.15)

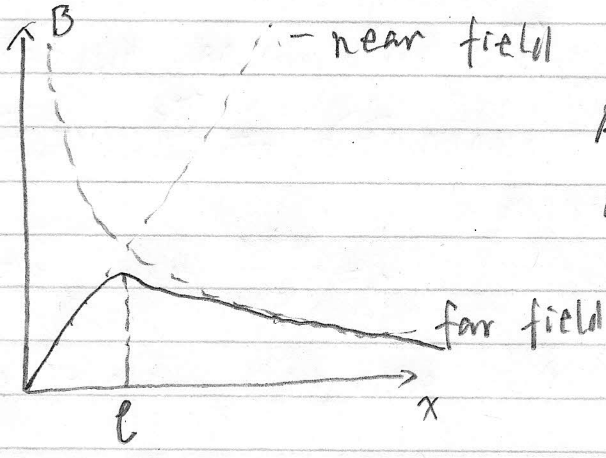
$$\Rightarrow \eta \frac{dB}{dx} \approx E \Rightarrow B \approx \frac{Ex}{\eta} : \text{ linear function}$$

If x is large, (3.15)

$$\Rightarrow \cancel{\eta \frac{dB}{dx} = -\frac{V_0 x}{a} B} \quad E - \frac{V_0 x}{a} B = 0$$

$$\Rightarrow B = \frac{Ea}{V_0 x} : \text{ hyperbola function}$$

(Continued)



At l: $\vec{J}=0$, $\vec{B}$ max

At $x=0$, $\vec{J}$ max, $\vec{B}=0$

l: boundary of current sheet

half of the current sheet $l=\sqrt{\frac{2\eta a}{V_0}}$

For corona, $\eta = 1\, m^2 s^{-1}$, $a = 10^4$ km, $V_0 = 10$ km/s

$$l = \sqrt{\frac{2 \cdot 1 \cdot 10^4 \times 10^3}{10 \times 10^3}} = \sqrt{2000} \approx 45 \text{ m}$$

Therefore, current sheet is very thin

\# plasma pressure P from MHD solution

From momentum equation

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \underbrace{\vec{j} \times \vec{B}}_{} \qquad \frac{1}{\mu}(\nabla \times \vec{B}) \times \vec{B}$$

$$\cancel{\rho \frac{\partial \vec{v}}{\partial t}} + \rho(\vec{v}\cdot\nabla)\vec{v} = -\nabla p + \underbrace{\frac{(\vec{B}\cdot\vec{\nabla})\vec{B}}{\mu_0}}_{=0} - \frac{1}{2}\frac{\nabla B^2}{\mu}$$

$\vec{B}$ only in $\hat{y}$

$$\Rightarrow \rho(\vec{v}\cdot\nabla)\vec{v} = -\nabla p - \nabla\left(\frac{B^2}{2\mu}\right)$$

$$\underbrace{\cancel{\vec{v} \times (\nabla \times \vec{v})}}_{=0 \text{. two-D field}} = \frac{1}{2}\nabla v^2 - (\vec{v}\cdot\nabla)\vec{v}$$

Vector identity
$$\vec{A} \times (\nabla \times \vec{B}) = \vec{A} \cdot \nabla \vec{B} - (\vec{A}\cdot\nabla)\vec{B}$$
$$\vec{A} \times (\nabla \times \vec{A}) = \vec{A} \cdot \nabla \vec{A} - (\vec{A}\cdot\nabla)\vec{A}$$
$$\nabla A^2 = \nabla(\vec{A}\cdot\vec{A}) = \vec{A}\cdot\nabla\vec{A} + \vec{A}\cdot\nabla\vec{A} = 2\vec{A}\cdot\nabla\vec{A}$$
$$\Rightarrow \vec{A} \times (\nabla \times \vec{A}) = \frac{1}{2}\nabla A^2 - (\vec{A}\cdot\nabla)\vec{A}.$$

(continued) $\nabla\left(p + \frac{B^2}{2\mu} + \frac{1}{2}\rho v^2\right) = 0$

thermal pressure — magnetic pressure — ram pressure (~~limit~~ inertial term)

At the center, $\vec{v} = 0$, stagnation point

$\vec{B} = 0$, null point

$P = P_e$: location of highest thermal pressure

$$P = P_e - \frac{1}{2}\rho v^2 - \frac{B^2}{2\mu}$$

$P \geq 0$, put the limit on the annihilation rate.

At $x = l$, B_{max}, $P_{min} > 0$

$$M_e < \frac{1.7(1+\beta_e)}{R_{me}}$$

$M_e = \frac{v_e}{v_{Ae}}$ Alfven Mach number:

a measure of energy dissipation rate

v_e: the flow velocity at $x = L_e$,

L_e: the characteristic size of the system

v_{Ae}: external Alfven speed $v_A = \sqrt{\frac{B^2}{\mu_0 \rho}}$

$\beta_e = \frac{P_e}{\frac{B_e^2}{2\mu}} = \frac{\text{Thermal pressure}}{\text{magnetic pressure}}$: plasma β of external field.

Because in corona, $\beta_e < 1$, $R_{me} = 10^6$

$M_e < 10^{-6}$, extremely slow, can't explain flares

Oct. 21. 2010

Steady reconnection : Classical Solution (CH4)

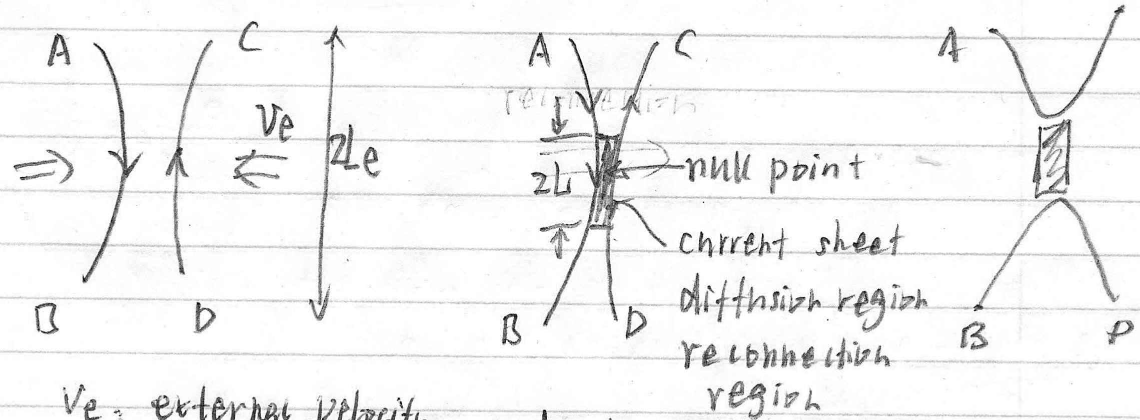
* Introduction to magnetic reconnection (CH4.1)
* Sweet-Parker Mechanism (CH4.2)
* Petschek's mechanism (CH4.3)

Introduction (CH 4.1)

To explain solar flares, a fast magnetic energy dissipation mechanism is needed,

It depends how fast the magnetic field can be taken into the diffusion region

Reconnection is found to be faster than pure diffusion



V_e: external velocity

L_e: external size

L: length of reconnection region

magnetic reconnection rate: V_e

Dimensionless magnetic reconnection rate: $M_e = \frac{V_e}{V_{Ae}}$

The goal is to find solution of high M_e ↑

(continued):

classical magnetic reconnection mechanism: always assumes a two-dimensional steady state

$$\vec{V} = (V_x, V_y, 0), \quad \vec{B} = (B_x, B_y, 0), \quad \vec{E} = (0, 0, E_z)$$

E_z : constant

$$\vec{E} + \vec{V} \times \vec{B} = \frac{\vec{J}}{\sigma}$$

$\Rightarrow$ $\vec{E}$ is constant, and only in z direction.

$\vec{J} = \frac{\nabla \times \vec{B}}{\mu}$: Large $\vec{j}$ only in current sheet and only $\hat{z}$ direction

magnetic dissipation time scale:

$$\tau_d = \frac{L^2}{\eta} = 10^{-9} L^2 T^{3/2}$$

In corona $T = 10^6$ K, $\eta = 1 \, m^2 s^{-1}$

If $\tau_d = 100$ sec. $L = 10$ m.

How to achieve such a thin current sheet?

To explain flares: (1) Large inflow velocity V_e.

(2) extremely thin current sheet