CSI 769 Fall 2010
Solar and Heliospheric Physics
Mid-term Exam
Nov. 4, 2010

Answer three and only three of the following questions. You have to indicate which one is omitted by you.

1. **Force Free Parameter α**
   (1) What is a force-free magnetic field?
   (2) Force-free parameter is defined as
   $$\nabla \times \vec{B} = \alpha(\vec{r})\vec{B}$$
   Prove that **α** is not completely arbitrary: it is a constant along a magnetic field line. This is to prove the following expression
   $$\vec{B} \cdot \nabla \alpha = 0$$

2. **Magnetic Induction Equation**
   a. Write down the generalized Ohm's law
   b. Start from Ohm's law and combine with Maxwell equations, derive the magnetic induction equation
   $$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{V} \times \vec{B}) + \eta \nabla^2 \vec{B}$$
   where $\eta = \frac{1}{\mu_0 \sigma}$

3. **Null Point, current sheet and separatrix**.
   Sketch the 2-D magnetic field lines described as follow, indicating the positions of two null points, one current sheet and four separatrix in your drawing.
   $$By + iBx = \frac{(z^2+1)}{(z^2-1)^{1/2}}$$
   where z=x+iy

4. **2-D Field**
   Consider a 2-D field, in which both magnetic field **B** and flow field **V** have only X and Y components. Start from the generalized Ohm's law, prove that (1) electric field **E** has only **Z** component, and (2) **E** is constant across the **X-Y** plane.

Useful Identities:
$$\nabla \cdot (\alpha \vec{A}) = \alpha(\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla \alpha$$
$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$