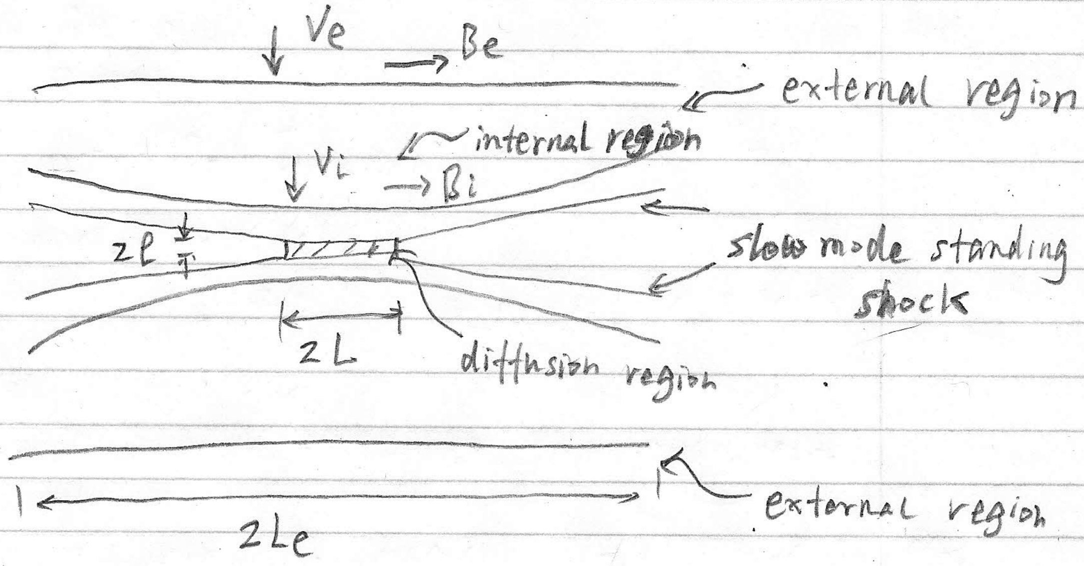
HW #8

1.




$V_e$: plasma velocity in the external region

$B_e$: magnetic field " " " "

$L_e$: geometric size of " " "

$V_i$: plasma velocity in the internal region

$B_i$: magnetic field " " " "

$L$: Length of the diffusion region

$l$: width of the diffusion region

Schematic of Petschek Reconnection Model

2. Magnetic Reconnection Rate.

Use MKS unit

Given: $B = 1000\ G = 10^{-1}$ tesla

$L_e = 10^5\ km = 10^8\ m$

$\eta = 1\ m^2/s$

$n_e = 10^9\ cm^{-3} = 10^{15}\ m^{-3}$ : number density

$\rho = n_e m_p = 10^{15} \times 1.67 \times 10^{-27} = 1.67 \times 10^{-12}\ kg/m^3$

— mass density

(1) $$V_A = \frac{B_e}{\sqrt{\mu_0 \rho}} = \frac{10^{-1}\ T}{\sqrt{4\pi \times 10^{-7}\ Hm^{-1} \cdot 1.67 \times 10^{-12}\ kg/m^3}} = \boxed{6.90 \times 10^7\ m/s}$$

or ~ 69000 km/s

(2) $$R_m = \frac{L V_A}{\eta} = \frac{10^8 \cdot 6.9 \times 10^7}{1} = \boxed{6.90 \times 10^{15}}$$

— dimensionless

(3) For 1-D stagnation flow model

$$M_e \equiv \frac{V_e}{V_A} = \frac{1}{R_m} = \frac{1}{6.90 \times 10^{15}} = \boxed{1.45 \times 10^{-16}}$$

Inflow speed : $V_e = M_e \cdot V_A = 1.45 \times 10^{-16} \cdot 6.90 \times 10^7$

$$V_e = \boxed{1.0 \times 10^{-8}\ m/s}$$

(4) For Sweet-Parker mechanism,

$$M_e \equiv \frac{V_e}{V_A} = \frac{1}{\sqrt{R_m}} = \frac{1}{\sqrt{6.90 \times 10^{15}}} = \boxed{1.20 \times 10^{-8}}$$

Inflow speed $V_e = M_e \cdot V_A = 1.20 \times 10^{-8} \cdot 6.90 \times 10^7 = \boxed{0.83\ m/s}$

(5) For Petschek mechanism

$$M_e \equiv \frac{V_e}{V_A} = \frac{\pi}{8 \log R_m} = \frac{\pi}{8 \log(6.90 \times 10^{15})}$$

natural log : ln

$$\boxed{M_e = 2.48 \times 10^{-2}}$$

$$V_e = M_e \cdot V_A = 2.48 \times 10^{-2} \cdot 6.90 \times 10^{7} = \boxed{1.71 \times 10^{6}\ m/s}$$

or $V_e = 1.71 \times 10^{3}\ km/s$ . => fast reconnection.

3. Petschek Reconnection Model

(1) From question 2.

$$V_{Ae} = 6.90 \times 10^{7}\ m/s$$

$$R_{me} = 6.90 \times 10^{15}$$

$$M_e = 2.48 \times 10^{-2}$$

$$V_e = 1.71 \times 10^{6}\ m/s$$

$$L_e = 10^{8}\ m$$

(2) $B_i = \frac{B_e}{2} = \frac{0.1}{2} = \boxed{5.0 \times 10^{-2}\ T}$

$V_i B_i = V_e B_e = E$ : E constant in 2-D flow

$$V_i = \frac{B_e}{B_i} V_e = 2 \cdot 1.71 \times 10^{6} = \boxed{3.42 \times 10^{6}\ m/s}$$

$$\frac{M_i}{M_e} = \frac{B_e^2}{B_i^2} = 4 \Rightarrow M_i = 4 M_e = \boxed{9.92 \times 10^{-2}}$$

To find L : ① find $R_{mi}$ first,

From Sweet-Parker. $M_i = \frac{1}{\sqrt{R_{mi}}}$

$$\Rightarrow R_{mi} = \frac{1}{M_i^2} = \frac{1}{(9.92 \times 10^{-2})^2} = 1.02 \times 10^{2}$$

And $R_{mi} = \frac{L V_{Ai}}{\eta}$ , $V_{Ai} = V_{Ae} \cdot \frac{B_i}{B_e} = \frac{V_{Ae}}{2} = 3.45\times10^{7}$ m/s

$$L_i = \frac{\eta R_{mi}}{V_{Ai}} = \frac{1.02\times10^{2}}{3.45\times10^{7}} = \boxed{2.95\times10^{-6}\ \text{m}}$$

or, (2) use the formula

$$\frac{L}{L_e} = \frac{1}{R_{me}} \cdot \frac{1}{M_e^{1/2}} \cdot \frac{1}{M_i^{3/2}}$$

$$L_i = 10^{8} \cdot \frac{1}{6.90\times10^{15}} \cdot \frac{1}{(2.48\times10^{-2})^{1/2}} \cdot \frac{1}{(9.92\times10^{-2})^{3/2}}$$

$$\boxed{L = 2.95\times10^{-6}\ \text{m}}$$

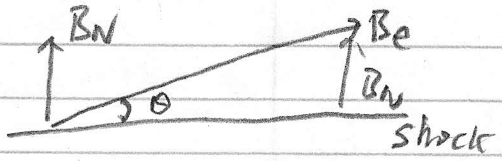
(3) $\frac{l}{L} = \frac{1}{\sqrt{R_{mi}}} = M_i$ : the Sweet-Parker

$$l = L M_i = 2.95\times10^{-6} \cdot 9.92\times10^{-2} = \boxed{2.93\times10^{-7}\ \text{m}}$$

(4) slow-mode shock speed $V_s$ equals the inflow speed of plasma in the external region, to form the standing shock

$$V_s = V_e = 1.71\times10^{6}\ \text{m/s}$$

(5) shock speed is the Alfven speed of the normal component




$$V_s = \frac{B_N}{\sqrt{\mu_0 \rho}}$$

$$B_N = 1.71\times10^{6} \cdot \sqrt{4\pi\times10^{-7} \cdot 1.67\times10^{-12}} = 2.48\times10^{-3}\ \text{T}$$

$$\sin\theta = \frac{B_N}{B_e} = \frac{2.48\times10^{-3}}{0.1} = 2.48\times10^{-2}.$$

$$\theta = 2.48\times10^{-2}\ \text{rad} = \boxed{1.42^{\circ}}$$