CSI 769 Fall 2010
Solar and Heliospheric Physics

Homework Assignment 7 - Solution

Assignment Date: Nov. 14, 2010
Due Date: Nov. 18, 2010

1. **MDH Waves**

Draw the Friedricks diagram of MHD wave to scale in the Sun's corona using the following parameters. The magnetic field is 1000 Gauss, temperature is 1 MK, and electron density is 10^9 /cm^3.

(1) What is the speed of Alfven wave?
(2) What is the speed of sound wave?
(3) What is the speed of the fastest MHD wave? What is its mode?
(4) What are the speed of slow mode, intermediate mode and fast mode waves when the propagation direction of the wave is 45° from the direction of magnetic field?
(5) Draw the Freidricks diagram using true numbers?

Note: (1) For the drawing, you need to use computer program to plot out the diagram, either in IDL or Maltab, or any other programs you prefer.
(2) The fast/slow mode MHD wave equation is found (from MHD equations using perturbation method) to be

$$\frac{\omega^2}{k^2} = \frac{V_A{}^2 + C_s{}^2}{2} \pm \frac{1}{2}\sqrt{(V_A{}^2 - C_s{}^2)^2 + 4C_s{}^2 V_A{}^2 \sin^2\theta}$$

(3) For a fully ionized plasma such as corona, thermal pressure $P=2n_ekT$
(4) For a fully ionized plasma such as corona, mass density $\rho = n_e m_p$

1. Solution
(1)

$$V_A = \sqrt{\frac{B^2}{\mu_0 \rho}} \text{ in SI unit; } \quad V_A = \sqrt{\frac{B^2}{4\pi\rho}} \text{ in cgs unit}$$

I will use SI unit in following calculations

$$B = 1000\ G = 0.1\ \text{Tesla}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \text{Henry m}^{-1}$$

$$n_e = 10^9\ \text{cm}^{-3} = 10^{15}\ \text{m}^{-3}$$

$$\rho = n_e m_p = 10^{15} \times 1.673 \times 10^{-27} = 1.673 \times 10^{-12}\ \text{kg m}^{-3}$$

$$V_A = \sqrt{\frac{(0.1)^2}{4\pi \times 10^{-7} \times 1.673 \times 10^{-12}}} = 6.90 \times 10^7\ \text{m s}^{-1}$$

In km/s, Alfven speed is ~ 69,000 km/s

(2)

$$C_s = \sqrt{\frac{\gamma P}{\rho}}$$

$$P = 2 n_e k T = 2 \times 10^{15} \times 1.38 \times 10^{-23} \times 1.0 \times 10^6 = 2.76 \times 10^{-2}\ Nm^{-2}$$

$$\rho = n_e m_p = 10^{15} \times 1.673 \times 10^{-27} = 1.673 \times 10^{-12}\ \text{kg m}^{-3}$$

$$C_s = \sqrt{\frac{5/3 \times 2.76 \times 10^{-2}}{1.673 \times 10^{-12}}} = 1.66 \times 10^5\ \text{m s}^{-1}$$

In km/s, sound speed is ~166 km/s,

which is much smaller than the Alfven speed.

(3)

The fastest MHD wave occurs when, according to the wave dispersion function, the sign is positive and the angle is 90 deg. The mode is fast mode, in which the magnetic pressure and thermal pressure couple with and strengthen each other. In this case, the formula can be written as

$$\frac{\omega^2}{k^2} = V_A{}^2 + C_s{}^2$$

$$V_{ph} = \frac{\omega}{k} = \sqrt{V_A{}^2 + C_s{}^2} = 6.90 \times 10^7\ \text{m s}^{-1}$$

(4)

$\theta = 45^0$

Slow mode : the sign is negative in the dispersion function

Fast mode : the sigh is positive in the dispersion function

Intermediate mode :

$V_{ph} = V_A \cos\theta.$

Plug in the numbers ==>

Slow mode : $\mathrm{V_{ph}} = 1.17 \times 10^5$ m s^{-1}

Intermediate mode : $\mathrm{V_{ph}} = 4.88 \times 10^7$ m s^{-1}

Fast mode : $\mathrm{V_{ph}} = 6.90 \times 10^7$ m s^{-1}

(5) Draw the Freidricks diagram. The two drawings are from Patrick Dandenault