HW #4 Solution

1. Magnetic Reynolds number. (50pts)

(1) According to PF (1-13) and (1-14)

$$\eta = \frac{c^2 e^2 m_e^{\frac{1}{2}}}{3(2\pi)^{3/2}\varepsilon_0} \ln\Lambda \, (k_B T_e)^{-\frac{3}{2}}$$

$$\eta = 1.05 \times 10^8 \, T_e^{-\frac{3}{2}} \ln\Lambda \ \ m^2 s^{-1}$$

$$\ln\Lambda = \begin{cases} 16.3 + \frac{3}{2}\ln T - \frac{1}{2}\ln n & T < 4.2\times10^5 K \\ 22.8 + \ln T - \frac{1}{2}\ln n & T > 4.2\times10^5 K \end{cases}$$

In the photosphere

$$\ln\Lambda = 16.3 + \frac{3}{2}\ln(5700) - \frac{1}{2}\ln(10^{21}) = 5.10$$

$$\eta = 1.05\times10^8 \, (5700)^{-\frac{3}{2}} \cdot 5.10$$

$$\eta = 1.24\times10^3 \ m^2 s^{-1} \quad : \text{diffusivity}$$

$$R_m = \frac{L_0 V_0}{\eta} = \frac{10^4 \, km \cdot 10^3 m/km \cdot 2 \, km/s \cdot 10^3 m/km}{1.20\times10^3}$$

$$R_m = 1.61\times10^7 \quad : \text{Magnetic Reynolds number}$$

$$\tau_d = \frac{L_0^2}{\eta} = \frac{(10^7)^2}{1.20\times10^3} = 8.06\times10^{10} \, s \simeq 2500 \text{ years}$$

Diffusion Time

(2) In the corona

$$\ln\Lambda = 19.35$$

$$\eta = 2.03 \ m^2 s^{-1}$$

$$R_m = 4.93\times10^{10}$$

$$\tau_d = 4.93\times10^{13} \, s \simeq 1.5 \text{ million yrs}$$

Hw #4 (continued)

2. 1-D Magnetic Diffusion (50 pts)

Diffusion equation: $\frac{\partial \hat{B}(x,t)}{\partial t} = \eta \frac{\partial^2 B(x,t)}{\partial x^2}$

(1) Prove $B(x,t) = \frac{2B_0}{\sqrt{\pi}} \operatorname{erf}\left(\frac{x}{\sqrt{4\eta t}}\right)$ is a solution

$$\operatorname{erf}(x) = \int_0^x e^{-u^2} du$$

First $\frac{\partial B(x,t)}{\partial t} = \frac{2B_0}{\sqrt{\pi}} \frac{\partial}{\partial t} \int_0^{\frac{x}{\sqrt{4\eta t}}} e^{-u^2} du$

Use the identity: $\frac{d}{dt}\int_0^{b(t)} f(x)\,dx = \frac{db(t)}{dt} f(b(t))$

$$\Rightarrow \frac{\partial}{\partial t}\int_0^{\frac{x}{\sqrt{4\eta t}}} e^{-u^2} du = \frac{x}{\sqrt{4\eta}}\left(-\tfrac{1}{2}\right) t^{-\frac{3}{2}}$$

$$\Rightarrow \frac{\partial B(x,t)}{\partial t} = -\frac{B_0}{\sqrt{\pi}} \frac{x}{\sqrt{4\eta}} \frac{1}{t^{\frac{3}{2}}} e^{-\left(\frac{x}{\sqrt{4\eta t}}\right)^2}$$

Second $\frac{\partial B(x,t)}{\partial x} = \frac{2B_0}{\sqrt{\pi}} \frac{\partial}{\partial x} \int_0^{\frac{x}{\sqrt{4\eta t}}} e^{-u^2} du = \frac{2B_0}{\sqrt{\pi}} \frac{1}{\sqrt{4\eta t}} e^{-\left(\frac{x}{\sqrt{4\eta t}}\right)^2}$

$$\frac{\partial^2 B(x,t)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial B}{\partial x}\right) = -\frac{B_0}{\sqrt{\pi}} \cdot \frac{4x}{(4\eta t)^{3/2}} e^{-\left(\frac{x}{\sqrt{4\eta t}}\right)^2}$$

$$\Rightarrow \frac{\partial B(x,t)}{\partial t} = \eta \frac{\partial^2 B(x,t)}{\partial x^2}$$

(2) $\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$

$$j = \frac{1}{\mu} \frac{\partial B(x,t)}{\partial x} = \frac{1}{\mu} \frac{2B_0}{\sqrt{\pi}} \frac{1}{\sqrt{4\eta t}} e^{-\left(\frac{x}{\sqrt{4\eta t}}\right)^2}$$

(3) (4). See the attachment. From the HW sheets of Pat Dandenault

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; CSI 769 Fall 2010 Semester
; Patrick Dandenault
; Homework #4, Problems 2.3 and 2.4
; October 21, 2010
;
PRO hw4

; define constants
n   = 1.0
B0  = 500
spi = SQRT(!pi)
u   = 4E-7*!pi
x   = FLTARR(100)

; define magnetic field and current density arrays
B1 = DBLARR(100) & B2 = B1 & B3 = B1
j1 = B1 & j2 = B1 & j3 = B1

; set up plotting device
DEVICE, DECOMPOSE=0
!P.CHARSIZE=1.25

; Problem 2.3 --------------------------------
; Calculate and plot the distribution of
; the magnetic field B(x,t).
FOR p = 0,99 DO BEGIN
  x(p)  = -50.0 + p
  B1(p) = (2*B0/spi) * ERF(x(p)/SQRT(4*n*0))
  B2(p) = (2*B0/spi) * ERF(x(p)/SQRT(4*n*10))
  B3(p) = (2*B0/spi) * ERF(x(p)/SQRT(4*n*100))
END
WINDOW,1
PLOT, x, B1, COLOR=0, BACKGROUND=255, THICK=2, $
      XTITLE='x', YTITLE='Magnetic Field B(x,t)', $
      TITLE='Magnetic Field B(x,t) vs. x for various t'
OPLOT, x, B2, LINESTYLE=1, COLOR=4, THICK=2
OPLOT, x, B3, LINESTYLE=5, COLOR=6, THICK=2
XYOUTS, 0.20, 0.80, 't = 0  SOLID LINE', /NORMAL, COLOR=0
XYOUTS, 0.20, 0.75, 't = 10 DOTTED LINE', /NORMAL, COLOR=0
XYOUTS, 0.20, 0.70, 't = 100 DASHED LINE', /NORMAL, COLOR=0

; Problem 2.4 ---------------------------------
; Calculate and plot the distribution of
; the electric current density j(x,t).
FOR p = 0,99 DO BEGIN
  j1(p) = (2*B0)/(u*SQRT(4*!pi*n*0)) * exp(-(x(p)/SQRT(4*n*0))^2)
  j2(p) = (2*B0)/(u*SQRT(4*!pi*n*10)) * exp(-(x(p)/SQRT(4*n*10))^2)
  j3(p) = (2*B0)/(u*SQRT(4*!pi*n*100)) * exp(-(x(p)/SQRT(4*n*100))^2)
END
WINDOW,2
PLOT, x, j2, COLOR=0, BACKGROUND=255, THICK=2, $
      XTITLE='x', YTITLE='Electric Current Density j(x,t)', $
      TITLE='Current Density j(x,t) vs. x for various t'
OPLOT, x, j3, LINESTYLE=5, COLOR=4, THICK=2
XYOUTS, 0.20, 0.75, 't = 10  SOLID LINE', /NORMAL, COLOR=0
XYOUTS, 0.20, 0.70, 't = 100 DASHED LINE', /NORMAL, COLOR=0
XYOUTS, 0.20, 0.80, 't = 0  NOT PLOTTED', /NORMAL, COLOR=0
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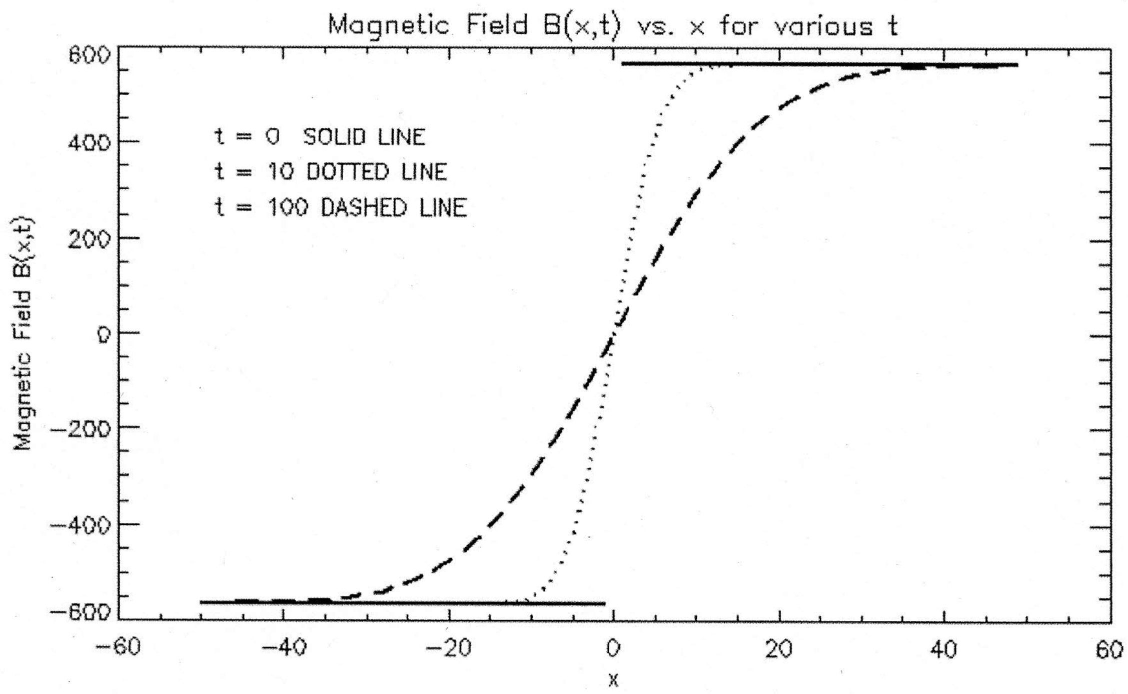
CSI 769 Homework 4 Patrick Dandenault October 21, 2010

Problem 2.3: Plot the Magnetic Field B(x,t)



Problem 2.4: Plot the Electric Current Density j(x,t)