HW #2 due . Sep 30, 2010

1. Unit system

Aschwanden book use cgs unit

Farraday's Law : $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$ (5.1.3)

Ampere's Law : $\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j}$ (5.1.4)

Priest - Forbes book use MKS, or SI unit

Farraday's Law $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ampere's Law, $\nabla \times \vec{B} = \mu \vec{j}$

The difference is caused by the usage of different unit systems

+ ~~Ampere~~ Farraday's Law from cgs to mks

$$\nabla \times \vec{E}(\text{statvol cm}^{-1}) = -\frac{1}{c}\frac{\partial \vec{B}(\text{Gauss})}{\partial t}$$

$$\nabla \times \vec{E}(\text{V m}^{-1})(3\times10^{4})^{-1} = -\frac{1}{c}\frac{\partial \vec{B}(\text{Tesla})(10^{+4})}{\partial t}$$

$$\nabla \times \vec{E}(\text{V m}^{-1}) = -\underbrace{\left(\frac{3\times10^{+4}}{10^{2}} \cdot \frac{10^{4}}{c}\right)}_{=1} \frac{\partial \vec{B}(\text{Tesla})}{\partial t}$$

$c = 3\times10^{8}$ m/s

$\Rightarrow$ $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ in MKS unit

* Ampere's Law from cgs to mks

$$\nabla \times \vec{B}(\text{Gauss}) = 4\pi \vec{j}\ (\text{statampere cm}^{-2})$$

$$\nabla \times [10^4 \vec{B}(\text{Tesla})] = 4\pi \left(\tfrac{1}{3}\times 10^{-5}\right)^{-1} \vec{j}\ (\text{A m}^{-2})$$

$$\nabla \times \vec{B}(\text{Tesla}) = \underbrace{\frac{4\pi \cdot 10^{-4}}{\left(\frac{1}{3}\times 10^{-5}\right)}} \vec{j}\ (\text{A m}^{-2})$$

In mks, $\mu = 4\pi \times 10^{-7}$ H m^{-1} or N A^{-2}

we used cgs -emu unit

now, try cgs-esu unit.

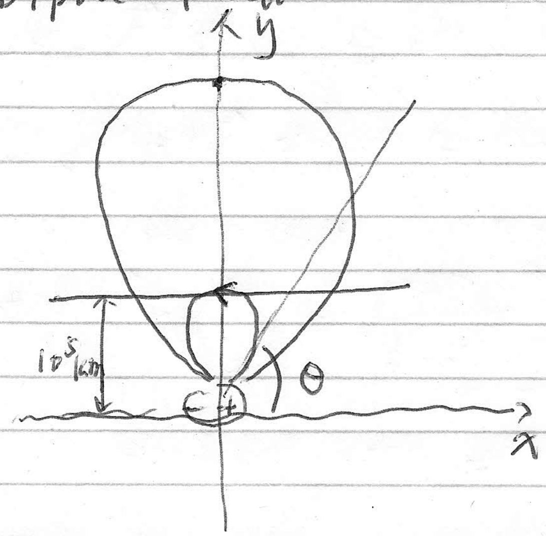
should use this, conversion between cgs-esu ⟷ mks

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j}, \quad \text{additional } c$$

$$\Rightarrow \nabla \times \vec{B}(\text{Tesla}) = \underbrace{\frac{4\pi \cdot 3\times 10^5 \cdot 10^{-4}}{3\times 10^8}}_{4\pi\times 10^{-7} = \mu} \cdot \vec{j}\ (\text{A m}^{-2})$$

$$\Rightarrow \nabla \times \vec{B}(\text{Tesla}) = \mu \vec{j} \quad \text{in MKS}$$

2. Dipole field



(1) Right above the dipole, $\theta = \frac{\pi}{2}$

$$B_r = 0$$

$$B_\theta = \frac{m \cdot \sin\frac{\pi}{2}}{r^3} = 1000 \text{ G} \quad (= B_{total})$$

$$m = B_\theta \cdot r^3 = 1000 \cdot (10^5 \text{ km})^3 = 10^3 \cdot (10^{10} \text{ cm})^3$$

$$\boxed{m = 10^{33} \text{ Gauss cm}^3}$$

(2) $$B_r = \frac{2m\cos(\frac{\pi}{2})}{r^3} = 0$$

$$B_\theta = \frac{m \sin\frac{\pi}{2}}{r^3} = \frac{10^{33} \cdot 1}{[(3+1)\times 10^5 \text{ km}]^3} = \frac{10^{33}}{(4\times 10^{10} \text{ cm})^3}$$

$$B_\theta = 15.6 \text{ G}$$

$$B_{total} = \sqrt{B_r^2 + B_\theta^2} = B_\theta = \boxed{15.6 \text{ G}}$$

(3) at $r = 3 \times 10^5$ km, $\theta = \frac{\pi}{4}$

$$B_r = \frac{2m\cos\theta}{r^3} = \frac{2 \cdot 10^{33} \left(\cos\frac{\pi}{4}\right)}{\left(3 \times 10^5 \text{ km} \times \frac{10^5 \text{ cm}}{1 \text{ km}}\right)^3}, \quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$B_r = 52.4 \text{ G}$$

$$B_\theta = \frac{m\sin\theta}{r^3} = \frac{10^{33} \cdot \sin\frac{\pi}{4}}{(3 \times 10^{10})^3} = 26.6 \text{ G}$$

From polar ~~angle to~~ coordinate to Cartisian Coor.

$$\begin{cases} B_x = B_r\cos\theta - B_\theta\sin\theta \\ B_y = B_r\sin\theta + B_\theta\cos\theta \end{cases}$$

$$\Rightarrow \begin{cases} B_x = 52.4\cos\frac{\pi}{4} - 26.6\sin\frac{\pi}{4} = \boxed{18.5 \text{ G}} \\ B_y = 52.4\sin\frac{\pi}{4} + 26.6\cos\frac{\pi}{4} = \boxed{55.9 \text{ G}} \end{cases}$$